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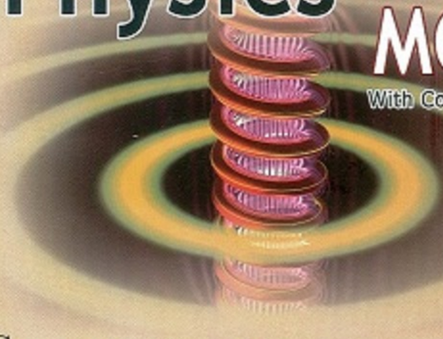

Sajid Iqbal

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Caravan's

PHYSICS

LECTURE GUIDE

(Test Your Physics)

for

Graduate and Post-graduate Classes and Competitive Examinations

By

ENGR. SAJID IQBAL

M.Sc. (Electrical Engineering)

Department of Electrical Engineering,

University of Gujrat

CARAVAN BOOK HOUSE

Kachehri Road, Anarkali Lahore.

Phones: 042-37212091, 37122955, 37352296

e-mail: caravanlhr@hotmail.com

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Chapter 1

MECHANICS

I. UNITS, DIMENSIONS & VECTORS

UNITS, DIMENSION AND ERRORS

Concepts and Equations

In physics, we deal with observations, measurement and description of natural phenomenon related to *matter* and *energy*. The natural phenomena may be classified as mechanics, properties of matter, sound, thermodynamics, light, electricity, atomic physics, nuclear physics, particle physics, semiconductors, superconductors and so on.

Physical Quantities

These are quantities that are used to describe the laws of physics. Physical quantities may be divided into six categories.

1. Constant or Ratio

Such quantities have only magnitude, for example, refractive index, dielectric constant and specific gravity. Such quantities have no units.

2. Scalars

These quantities have magnitude and unit. Some of them may have direction also but vector laws are not applied. Examples are charge, mass, distance, speed and current.

3. Vectors

These quantities possess magnitude, unit and direction. They also follow triangle law of addition. For example, velocity, force, momentum, torque and displacement.

4. Phasors

These possess magnitude (amplitude) and phase. Thy follows triangle law. Simple Harmonic Motion (SHM), waves, AC voltage and AC current are phasors.

5. Tensors

Such quantities do not have any specified direction but have different values in different directions. For example, moment of inertia. In aniso torpic media even density, refractive index, dielectric constant, electric conductivity, stress, strain and so on become tensor. A physical quantity which has only one component is called a scalar or a tensor of zero rank. If it has more than one component but less than or equal to four, it is called a vector or tensor of rank 1. If the components are greater than four, it is termed as tensor of a higher rank.

6. Conversion Factors

Some physical quantities convert into another when multiple by a factor. For example, in a wave $y = y_0 \sin(\omega t - kx)$, k is a conversion factor. When k is multiplied by displacement or path difference it generates phase or phase difference. In frequency modulation $kE_m f_c = \delta$, k converts voltage into angle and is termed as a conversion factor. Many other conversion factors can be thought of.

In general, a physical quantity = magnitude $\times$ unit. If the unit changes, the magnitude will also change. We apply $n_1 u_1 = n_2 u_2$..

Physical quantities may be divided into fundamental and derived quantities.

Fundamental Quantities

The quantities that do not depend upon any other quantity are called "fundamental or absolute or basic quantities". Initially, only three fundamental quantities — length, mass and time — were considered. With the development of science, four more physical quantities were added. These are temperature, electric current, luminous intensity and amount of a substance.

Derived Quantities

The physical quantities derived form fundamental quantities are called derived quantities like velocity, acceleration, force and momentum.

Units

The fixed and definite quantity taken as standard of reference with which other quantities of the same kind are measured is defined as a "unit".

Fundamental Units

The units of fundamental quantities are called "fundamental units". For example, units of length, mass and time or those of fundamental physical quantities, are called fundamental units. Table 1.1 lists all fundamental and supplementary units and their symbols.

Derived Units

Units of derived physical quantities are called "derived units". For example, units of velocity, density, force, momentum and volume.

Initially, three system of units, namely, CGS, FPS and MKS based on three fundamental quantities, length, mass and time, came into existence. In 1970, in a world confidence a consensus evolved and a standard international system of units was developed. It is more popularly known as the SI system. In addition to seven fundamental units,, two supplementary units were also included, namely, plane angle or angle (unit radian abbreviated as rad) and solid angle (unit stredian abbreviated str). SI units since 1978 are observed throughout the world.

Practical Units

Apart from fundamental units, supplementary units and derived units, we come across some practical units like light year, horse power, energy unit (1 unit = 3.6×10^6 J)Curie (Ci), Rutherford (R) etc.

While writing a unit, the following convention is adopted:

(a) Unit named after a person stars with a capital letter. For example, Newton is written as N, Curie as Ci and Rutherford as R.

(b) Fundamental units are written with small letters, for example, metre as m and kilogram as kg.

(c) The symbols are not expressed in plural form. For example, 50 metres will be written as 50 m.

(d) Punctuation mark such as full stop are not used after the symbol of unit. For example, 1 Milliliter = 1 ml or 1 cc (not m.l. or c.c.)

Table 1.1 Fundamental Physical Quantities

Physical quantity	SI Unit	Dimensional Symbol	Unit Symbol
Length	metre	L	m
Mass	kilogram	M	kg
Time	second	T	s
Temperature	Kelvin	K	K
Electric current	Ampere	A	A
Luminous Intensity	Candela	I	Cd
Amount of substance	mole	mol	mol

Physical quantity	SI Unit	Dimensional Symbol	Unit Symbol
Supplementary units			
Angle	radian	--	rad
Solid angle	stredian	--	str

Standards of Length

The most common unit is meter (m), Foot is also used sometimes. In 1889 the standard meter was defined as the distance between two fixed marks engraved on a platinum-iridium bar preserved at constant temperature of 73.16 K and constant pressure of 1 bar in the international bureau of weights and measures at Serves near Paris in France. All other meters are calibrated with it to an accuracy of 0.1 ppm.

In 1960 the standard metre was defined in terms of wavelength of Kr-86 and is called atomic standard of length. 1 m is the distance covered by 1650763.73 wavelength of orange red light of Kr-86 in vacuum. An accuracy $1:10^9$ parts can be obtained with it.

In 1983 metre was defined as the length of path travelled by light in a vacuum in $\frac{1}{299,792,458}$ th second.

Some other important units of length are

$1\ \text{A} = 10^{-10}$ m,

1 x-ray unit (1 XU) 10^{-13} m

1 yard = 3 foot = 0.9144 m,

1"(inch) = 2.54 cm

1 astronomical unit (1 AU)

$= 1.49 \times 10^{11}$ m,

1 light year (1 ly) = 9.46×10^{15} m,

1 parsec (1 pc) = 3.08×10^{16} m

= 3.26 1y

Standard of Mass

Originally, 1 kg mass was defined as the mass of 1 litre (10^3 cc) of water at 4°C, nowadays standard kg is the mass of platinum-iridium cylinder stored in a special vault in the International Bureau of Standard at Serves (France). The accuracy of this standard is 1 in 10^8 parts.

To measure atomic masses, the unit amu (or u) is used.

$$1\ \text{u} = \frac{1}{12}\text{th of mass of } {}^{12}_{6}\text{C atom}$$

or $$1\ \text{u} = \frac{1}{12} \times \left(\frac{12}{6.023 \times 10^{26}}\right) \text{kg}$$

$$= 1.67 \times 10^{-27}\ \text{kg}$$

In FPS system pound (lb) is the unit of mass, sometimes **slug** is also used.

1 lb = 0.453592737 kg,

1 slug = 32.2 lb = 14.59 kg

N.B. In astrophysics, we sometimes come across Chandrashekhar limit. 1 Chandrashekhar limit = 1.4 times mass of the sun = 2.8×10^{30} kg. Chandrashekhar showed that if the mass of an object becomes 1.4 times the mass of the sun, under gravitational collapse it turns out to be white dwarf.

Standards of Time

Initially, it was defined on the basis of solar or lunar motion.

$$1\ \text{mean solar day} = \frac{1\ \text{year}}{365.25\ \text{days}}$$

and $$1\ \text{s} = \frac{1\ \text{year}}{365.25 \times 24 \times 60 \times 60}$$

Bu because of tidal friction, the length of a day is increasing at a rate of 7 μs per year. Therefore, in 1965 the atomic standard as define. According to this standard, 1s is the interval of 91192631770 vibrations of radiation corresponding to the transition between two specific hyperfine levels in ^{133}Cs (cesium) clock which will go wrong by 1s in 3000 years. Hydrogen MASER promises a producing error of 1s in 33,000,000 years.

N.B. (i) time can never flow back, i.e., negative time does not exist and (ii) at a given instant of time, a particle cannot be present in more than one position in space.

Table 1.2 describes prefixes used for multiple and submultiples of metric quantities.

Dimensions and Dimensional Formulae

All physical quantities can be expressed in terms of seven fundamental quantities. The powers to which these fundamental physical quantities be raised are termed as "dimensions". For example, force = MLT^{-2}, .i.e., force has dimensions of mass, length and time as 1, 1 and –2 and $[MLT^{-2}]$ is the dimensional formula for force. Dimensional formulae for work, torque and resistance are $[M^2LT^{-2}]$, $[ML^2T^{-2}]$ and $[ML^2T^{-3}A^{-2}]$ respectively.

Table 1.2 Standard Prefixes for the SI Units of Measure

Multiples			Fractions		
Name	Symbol	Factor	Name	Symbol	Factor
deca	da	10^1	deci	D	10^{-1}
hecto	h	10^2	centi	C	10^{-2}
kilo	k	10^3	milli	m	10^{-3}
mega	M	10^6	micro	M	10^{-6}
giga	G	10^9	nano	n	10^{-9}
tera	T	10^{12}	pico	p	10^{-12}
peta	P	10^{15}	femto	f	10^{-15}
exa	E	10^{18}	atto	a	10^{-18}
zetta	Z	10^{21}	zepto	z	10^{-21}
yotta	Y	10^{24}	yocto	y	10^{-24}

Application of Dimensional Formula

(a) To check the correctness of a given physical relation. It is based on the principle of homogeneity, that is, the dimensions on two sides for a given relation. For example,

$F = 6\pi\eta rv$

Dimensions of LHS $F = MLT^{-2}$

RHS $6\pi\eta rv = [ML^{-1}T^{-1}]\,[L]\,[LT^{-1}]$

$= MLT^{-2}$

LHS and RHS have identical dimensions; therefore, the relations is dimensionally correct.

If the dimensions on two sides differ, the relation is incorrect.

(b) To derive a relation. We illustrate with an example. The amount of liquid flowing per second through a tube of radius r depends upon radius r of the tube, coefficient of viscosity and pressure gradient. Derive the expressions.

$$\frac{dV}{dt} \propto \eta^a r^b \left(\frac{p}{l}\right)^c$$

or $$\frac{dV}{dt} = \eta^a r^b \left(\frac{p}{l}\right)^c$$

where k is a dimensionless constant.

Then $[M^0L^3T^{-1}]$

$= [ML^{-1}T^{-1}]^a [L]^b [ML^{-2}T^{-2}]^c$

$= M^{a+c}L^{-a+b-2c}T^{-a-2c}$

Comparing powers of M, L and T on both sides.

$0 = a + c$

$3 = -a + b - 2c$

$-1 = -a - 2$

Solving, we get a = –1, b = 4, c = 1

Thus $$\frac{dV}{dt} = k\,\frac{r^4 \frac{p}{l}}{\eta}$$

The value of k is determined experimentally.

Hence, $\frac{dV}{dt} = \frac{\pi r^4 P}{8\eta l}$; $k = \pi/8$

(c) To convert the value of a physical quantity form one system of units to another system of units. We illustrate with an example. Let us convert 1J (SI) to erg. (CGS) system energy

$E = ML^2T^{-2}$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]^2 = \left[\frac{T_1}{T_2}\right]^{-2}$$

$$= 1\left[\frac{10^3}{1}\right]\left[\frac{10^2}{1}\right]^2 = \left[\frac{1}{1}\right]^{-2}$$

$= 10^7$ erg

Limitations of Dimensional Analysis

1. Dimensional analysis cannot be applied to derive a relation involving sum of products or product of sums from a relation like $s = ut + \frac{1}{2}at^2$. It can derive a relation of a single product term. If $s = ut + \frac{1}{2}at^2$ is to be derived using dimensional analysis then $s = ut$ and $s = \frac{1}{2}at^2$ will be derived separately and then added from your knowledge.
2. Dimensional analysis cannot be applied to derive a relation involving more than three unknowns. However, we can join two variables to make one, for example, we used pressure gradient $\left(\frac{p}{l}\right)$ and not p and l separately. Note that we an check the correctness of a relation involving any number of variables or terms.
3. If two quantities have same dimensions then such a relation involving these quantities cannot be derived. However, we can check the correctness of the relation.
4. Numerical constants, trigonometric ratios and other dimensionless ratio cannot be derived.
5. For a given dimension, physical quantity is not unique. For example, Torque and energy have the same dimensional formula $[ML^2T^{-2}]$

Significant Figures

These give the accuracy with which a physical quantity is expressed. The number of digits which are known reliably or about which we have confidence in measurement, plus the first digit that is uncertain, are termed as "significant figures". For instance, length of a table is 137.2 cm. This has 4 significant figures and 2 (after decimal) is uncertain. It is worth mentioning that significant of a physical quantity depends upon the least count of the instrument with which it is being measured.

Rules for Determining Significant Figures

1. All the nonzero digits are significant, for example, 187.25 ha 5 significant digits.
2. All the zeros occurring between two nonzero digits are significant. For instance, 105.003 has 6 significant digits.
3. The zeros occurring between the decimal point and the non-zero digits are not significant provided integral part is zero, i.e. in 0.0023 there are only two significant point figures.
4. All zeros to the right of a non-zero digit in number written without decimal are not significant. For example, 32500 has only 3 significant figures.

Exception

This rules does not work when we record the values on actual measurement basis. For example, distance between two places is 121 m has 4 significant digits.

5. All zeros occurring to the right of non-zero digit in a number written with a decimal point are significant. For example, 2.3200 has 5 significant figures.

N.B. The number of significant figures does not vary with the choice of units. For example, length of a rod is 76 cm. If we the represent it as 0.76 m or 0.00076 km significant figures remain 2.

In exponent form, exponent figure do not contribute to significant figure. Thus, 7.24×10^7 has only 3 significant digits.

RULES FOR ROUNDING OFF

1. If the digit to be dropped is less than 5, the preceding digit be kept unchanged. For example, 3.92 may be expressed 3.9.
2. If the digit to be dropped is greater than 5, the preceding digit be increased by 1. For example, 5.87 be rounded off as 5.9.
3. If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit be increased by 1. For example, 14.454 be rounded off as 14.5 to the first decimal place.

Resolution, Accuracy and Precision an Instrument

Resolution

Stands for least count or the minimum reading which an instrument can read.

Accuracy

An instrument is said to be accurate if the physical quantity measured by it resemble very closely its true value.

Precision

An instrument is said to have high degree of precision if the measured value remains unchanged, howsoever, large number of times it may have been repeated.

Error in Measurement

Deviation between measured and actual (or mean) value of a physical quantity is called "error". For example, if a_m is measured value of a physical quantity and its true value is a_t then error $\Delta a = |a_m - a_t|$. Error may be divided into two types: systematic and random error.

Systematic Error

Errors arising due to the system of measurement or the errors made due to parts involved in the system of measurement are called "systematic errors". Since the system involves instrument, observer and environment, systematic error is of three types, namely, instrumentation error, personal error (error made by observer due to carelessness or eye defect) and environment error. Instrument error equal to least count of instrument is unavoidable and hence is always accounted for.

Random Error (or Statistical Error)

The error which creeps in due to a large number of events or large quantity termed as "random error". Consider an example. The probability of tossing a coin 1/2. if we loss a coin 1000 times or 1000 coins are tossed simultaneously then we will hardly ever get 500 heads and 500 tails. Thus we find even random errors cannot be removed.

VECTORS

Concepts and Equations

Directed segments (physical quantities having magnitude and direction) and which follow triangle law of addition are called "vectors". For example force, acceleration, torque, momentum, angular momentum etc.

Properties of Vectors

In addition to magnitude and unit (a) it has specified direction, (b) it obeys triangle law of addition, (c) their addition is commutative i.e.

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

(d) Their addition is associative

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A}\,(\vec{B} + \vec{C})$$

Representation of Vectors

Vectors may be represented in two forms: polar and Cartesian.

(fig.)

Fig. 1.1 Polar representation of a vector

Polar Form

In this form $\vec{OA} = (r, \theta)$ where r is magnitude and θ is angle as shown in Fig. 1.1.

Cartesian Form

In this form $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ where a_1, a_2 and a_3 are coefficient and $\hat{i}$, $\hat{j}$, $\hat{k}$ are unit Vectors along x, y and z directions, respectively, as illustrated in Fig. 1.2.

(fig.)

Fig. 1.2 Unit vector representation in rectangular coordinate system

Types of Vector

In general vectors may be divided into three types:

1. Proper Vectors
2. Axial Vectors
3. Inertial or Pseudo Vectors

Proper Vectors

Displacement, force, momentum etc. are Proper Vectors.

Axial Vectors

The vectors which act along axis of rotation are called "axial vectors". For example, angular velocity, torque, angular momentum, angular acceleration are axial vectors.

Pseudo or Inertial Vectors

The vectors used to make an on inertial frame of reference into inertial frame of reference are called pseudo or inertial. Vector may further be subdivided as:

(i) Null Vector

It has zero magnitude and indeterminate direction.

(ii) Unit Vector

Magnitude of unit vector is 1. It specifies direction only. Unit vector of a given vector is $\hat{a} = \frac{\vec{A}}{|\vec{A}|}$ i.e. vector divided by its magnitude represents unit vector.

(iii) Like Vector or Parallel Vectors

If two vectors have the same direction but different magnitude then they are said to be "parallel or like vectors". Fig. 1.3(a) shows like vectors.

(fig.)

Fig. 1.3(a) Like vectors

(iv) Unlike Vectors

Two factors having opposite directions and unequal magnitudes are called "unlike vectors or parallel vectors in opposite sense". If their magnitude are equal, they are called "opposite vectors". Fig. 1.3(b) shows unlike vectors.

(fig.)

Fig. 1.3(b) Unlike vectors

(v) Equal Vectors

Two parallel vectors having equal magnitudes are called "equal vectors".

(vi) Co-initial Vectors

If vectors have a common initial point, they are known as co-initial vectors. In Fig. 1.4 $\vec{OA}$, $\vec{OB}$ and $\vec{OC}$ are "co-initial vectors".

(fig.)

Fig. 1.4 Co-initial Vectors

(vii) Co-linear Vectors

Like, unlike, equal, opposite vectors may be grouped as "co-linear vectors" if they are either in the same liner or parallel.

(viii) Co-planar Vectors

Vectors lying in the same plane are termed as "co-planar".

Resolution of a Vector

Resolving a vector into its components is called "resolution of a vector". Using triangle law one can write in Fig. 1.5.

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

or $$\vec{A} = \vec{A}_x\,\hat{i} + \vec{A}_y\,\hat{j}$$

or $$\vec{A} = A\cos\theta\,\hat{i} + A\sin\theta\,\hat{j}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

and $$\tan\theta = \frac{A_y}{A_x}$$

or $$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

(fig.)

Fig. 1.5 Resolution of a Vector

Laws of additions of Vectors

Vectors can be added using

(a) Triangle Law

(b) Parallelogram Law

(c) Polygon Law

Triangle Law

If two vectors acting on a body may be represented completely (in magnitude and direction) by two sides of a triangle taken in order, then their resultant is represented by third side of the triangle taken in opposite directions. In the Fig. 1.6(a).

$$\vec{OP} + \vec{PQ} = \vec{OQ}$$

or $$\vec{A} + \vec{B} = \vec{R}$$

(fig.)

Fig. 1.6 Triangle law illustration

If three vectors acting on a body may completely be represented by three sides of a triangle taken in order then the system is in equilibrium. In Fig. 1.6(b)

$$\vec{OP} + \vec{PQ} + \vec{QR}$$

$$= \vec{A} + \vec{B} + (-\vec{R})$$

or $$\vec{R} - \vec{R} = 0$$

Parallelogram Law

If two vectors acting on a body may be represented completely by two adjacent sides of a parallelogram, then their resultant is represented by a diagonal passing through the common point. In Fig. 1.7 from equal or $\vec{PL} = \vec{OQ} = \vec{B}$ from Δlaw $\vec{A} + \vec{B} = \vec{R}$.

(fig.)

Fig. 1.7 Parallelogram law of vector illustration

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\sin\theta}$$

$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

N.B. $A - B \leq A + B$ i.e. $R_{min} = A - B$ when $\theta = 180°$ and $R_{max} = A + B$ when $\theta = 0°$ $0 \leq$

$\theta \le 180°$. Remember θ cannot exceed 180°.

N.B. Minimum number of coplanar vectors whose sum can be zero (or required for equilibrium)

= 2 (if vectors are equal and opposite)

= 3 if vectors are unequal or not opposite, minimum number of non-coplanar vectors whose sum can be zero = 4.

N.B. Subtraction of a vector is equivalent to addition of a negative vector.

Multiplication of Vector

Two types of multiplication is defined (a) dot product or scalar product, (b) cross product or vector product.

(a) Dot Product or Scalar Product

If the product of two vectors is a scalar, then this rule is applied $\vec{A} \cdot \vec{B} = AB\cos\theta$.

N.B. $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$, i.e., scalar product is commutative $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$, i.e., scalar product follows distributive law.

Rules

(i) $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

(ii) $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{i} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{j} = \hat{i} \cdot \hat{k}$

$= \hat{k} \cdot \hat{i} = 0$

Application of Dot Product

1. When the product of two vectors is a scalar. For example
   $W = \vec{F} \cdot \vec{s}$, $P = \vec{F} \cdot \vec{v}$
   current $I = \int \vec{j} \cdot \vec{ds}$,
   magnetic flux $\phi = \int B \cdot ds$ etc.
2. To find an angle between two vectors
   $\theta = \cos^{-1}\left(\frac{\vec{A} \cdot \vec{B}}{|A||B|}\right)$.
3. If the dot product of two non zero vectors is zero, then they are perpendicular to one another.
4. Find the component of a vector along a given direction. For instance, the component of $\vec{A}$ along $\vec{B}$ is $A\cos\theta = \frac{\vec{A} \cdot \vec{B}}{B}$.

(b) Cross Product or Vector Product

This product is used when the product of two vectors is a vector i.e., $\vec{A} \times \vec{B} = AB \sin\theta\ \hat{n}$ where $\hat{n}$ is a unit vector perpendicular to both A and B. Apply right-handed screw rule to find direction of $\hat{n}$ or $\vec{A} \times \vec{B}$. Vector product is noncommutative i.e. $\vec{A} \times \vec{B} \ne \vec{B} \times \vec{A}$ (magnitude will be equal but direction will be opposite). Vector product is distributive. i.e.,

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

Rules

(i) $\vec{A} \times \vec{A} = 0 = \hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k}$

(ii) $\hat{i} \times \hat{j} = \hat{k} = -\hat{j} \times \hat{i}$, $\hat{j} \times \hat{k} = \hat{i}$
$= -\hat{k} \times \hat{j}$, $\hat{k} \times \hat{i} = \hat{j} = -\hat{i} \times \hat{k}$

Application of Vector Product

1. Cross product is used in rotational motion or product of two vectors is a vector. For example,

   Torque $\vec{\tau} = \vec{r} \times \vec{F}$,

   Pointing vector $\vec{P} = \vec{E} \times \vec{H}$
   $= \frac{1}{\mu_0}(\vec{E} \times \vec{B})$,

   Angular momentum $\vec{L} = \vec{r} \times \vec{p}$, $\vec{v}$
   $= \vec{\omega} \times \vec{r}$, $F = q \cdot (\vec{v} \times \vec{B})$.
2. If the vector product of two nonzero vectors is zero, then they are parallel.

3. It can be used to find angle θ

$$\theta = \sin^{-1}\left[\frac{|\vec{A}\times\vec{B}|}{|\vec{A}|\,|\vec{B}|}\right]$$

4. $|\vec{A} \times \vec{B}|$ represents area of a parallelogram whose sides are A and B. $\frac{1}{2}\left|\vec{D}_1 \times \vec{D}_2\right|$ represents area of a parallelogram where D_1 and D_2 are diagonals of the parallelogram.

Sine $\vec{D}_1 = \vec{A} + \vec{B}$

and $\vec{D}_2 = \vec{A} - \vec{B}$

$\therefore$ Area of a ||gm =

$$\frac{1}{2}\left|(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})\right|$$

(fig.)

Fig. 1.8

5. $\frac{1}{2}\left|\vec{A} \times \vec{B}\right|$ represents area of a triangle when A and B are two sides of the triangle.

Relative Velocity

Since absolute rest or absolute motion do not exist, therefore, every motion is a relative motion. Though, for convenience, we assume earth at rest and in common language measure the speed or velocity with respect to ground. But if two bodies A and B are moving with velocities V_A and V_B then relative velocity of A with respect B may be thought of velocity of A by bringing B to rest by applying equal and opposite velocity of B. Alternatively, Vector law may be applied from Fig. 1.9.

$$\vec{V}_{AG} = \vec{V}_{AB} + \vec{V}_{BG}$$

$$\vec{V}_{AB} = \vec{V}_{AG} - \vec{V}_{BG}$$

$$\vec{V}_{AB} = \vec{V}_{A} - \vec{V}_{B}$$

(fig.)

Fig.1.9 Relative velocity illustration

The best way to solve the questions on relative velocity is to resolve it into x and y components, thus

$$\vec{V}_{AB} = (\vec{V}_{Ax}\hat{i} + \vec{V}_{Ay}\hat{j}) - (\vec{V}_{Bx}\hat{i} + \vec{V}_{By}\hat{j})$$

$$= (\vec{V}_{Ax} - \vec{V}_{By})\,\hat{i} + (\vec{V}_{Ax} - \vec{V}_{By})\,\hat{j}$$

Then $|V_{AB}| = \sqrt{(V_{Ax}-V_{Bx})^2+(V_{Ay}-V_{By})^2}$

and $\tan\beta = \frac{V_{Ax} - V_{By}}{V_{Ax} - V_{Bx}}$ with respect to x-axis or $\hat{i}$ direction $\tan\beta' = \frac{V_{Ax} - V_{By}}{V_{Ax} - V_{Bx}}$ with respect to y-axis or $\hat{j}$ direction.

II. NEWTON’S LAWS OF MOTION, GRAVITATION, WORK, POWER AND ENERGY

LINEAR MOTION

Concepts and Equations

Motion

A body in motion keeps changing its position with respect to its surroundings with the passage of time. If the body does not change its position with respect to time it is said to be at rest.

Frame of Reference

A set of coordinates x, y, z and t is said to be a "frame of reference". Frame of reference may be inertial or non-inertial. Inertial frame of reference is one which is either fixed or moves with a uniform velocity in the same straight line. Non-inertial or accelerated frame of reference has an acceleration 'a'. Newton's laws are valid only in inertial frame. Pseudo or inertial vectors are to be applied to make the frame of reference inertial from non-inertial so that Newton's laws may be applied.

One-Dimensional Motion

If the particle changes its position only in one of the x, y, or z directions with respect to time, then the motion is said to be "one-dimensional". Since the particle moves along a straight line, the motion may also be termed as "linear or rectilinear".

Speed

The time rate of change of distance is called "speed", that is, $v = \frac{dx}{dt}$ unit ms^{-1}.

Velocity

The time rate of change of displacement is called "velocity", that is, $\vec{v} = \frac{d\vec{x}}{dt}$. Unit ms^{-1}, cms^{-1} and ft s^{-1} in SI, CGS and FPS system, respectively, $v = LT^{-1}$.

Displacement

The shortest distance between initial and final position of the practice is called "displacement".

Acceleration

The time rate of change of velocity is called "acceleration" $\vec{a} = \frac{d\vec{v}}{dt}$ units is ms^{-2}, cm s^{-2} and ft s^{-2} in SI, CGS and FPS system, respectively, $a = LT^{-2}$. Speed, velocity or acceleration may be of four types. We define here velocity and others can be anticipated in similar terms.

(a) Instantaneous Velocity

The velocity, at a particular instant of time is called instantaneous velocity, for example, velocity, at 4.82 s may be expressed as $\vec{v} = \left.\frac{d\vec{x}}{dt}\right|_{=4.82\text{ s}}$

(b) Uniform Velocity

If $\frac{dx}{dt}$ constant throughout the motion and direction of motion does not vary throughout then such a velocity is called "uniform velocity" Fig. 1.10(a) shows displacement time graph and Fig. 1.10(b) shows velocity time graph for a uniform velocity.

(fig.)

Fig. 1.10 Uniform velocity

(c) Variable Velocity

If $\frac{dx}{dt}$ is not constant but varies at different intervals of time or $\frac{dx}{dt}$ is constant but direction varies or both vary, then such a velocity is said to be variable velocity. Fig 1.11(a) illustrates x – t graph for a body moving with variable velocity and Fig. 1.11(b) shows velocity Vs time variation for a body moving with variable velocity.

(fig.)

Fig. 1.11(a) Variable velocity

(d) Average Velocity

It is that uniform velocity with which if the body would have moved it would have covered the same displacement as it does otherwise by moving with variable velocity. Thus

$$v_{av} = \frac{\text{total displacement covered}}{\text{total time taken}}$$

Average Velocity in Different Cases

(i) **Particles covering different displacement in different times**. Assume a particle covers s_1 displacement in t_1 and s_2 in time t_2 an so on then average velocity is

$$v_{av} = \frac{s_1 + s_2 + s_3 + \ldots}{t_1 + t_2 + t_3}$$

$$= \frac{s_1 + s_2 + s_3 + \ldots}{\frac{s_1}{v_1} + \frac{s_2}{v_1} + \frac{s_3}{v_3} + \ldots}$$

Special case if $s_1 = s_2 = s$.

$$v_{av} = \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2v_1v_2}{v_1 + v_2}$$

(harmonic mean)

(ii) **Bodies moving with different velocity in different intervals of time**. A body moves with velocity v_1 in time t, v_2 in time t_2 and so on then v_{av} is given by

$$v_{av} = \frac{v_1t_1 + v_2t_2 + \ldots}{t_1 + t_2 + \ldots}$$

Special case if $t_1 = t_2 = t_3 = \ldots t_n = t$

Then $v_{av} = \frac{v_1 + v_2 + \ldots + v_n}{n}$

(Arithmetic mean)

Equation of Motion

(a) $v = u + at$

(b) $s = ut + \frac{1}{2}at^2$

(c) $v^2 - u^2 = 2as$

(d) $s_{nth} = u + \frac{8}{2}(2n - 1)$

The condition under which these equations can be applied

1. Motion should be 1-dimensional.
2. Acceleration should be uniform.
3. Frame of reference should be inertial.

White drawing graph compare your equation with the following and then draw (matching the equation) graphs.

1. y = mx + c straight line passing with positive intercept on y-axis.

   y = mx straight line passing through origin.

   y = mx - c straight line with negative intercept (on y-axis).

(fig.)

Fig. 1.12 Straight Lines

2. $x^2 + y^2 = a^2$ circle with centre at origin. $(x - h)^2 + (y - k)^2 - r^2$ a circle with centre at (h, k).
3. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ represents ellipse.
4. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ shows hyperbola.
5. $y = \frac{1}{x}$ or xy = k represents a rectangular hyperbola. See Fig. 1.13(a).
6. $y = y_0\ e^{-ax} = y = y_0\ (1 - e^{-ax})$ represents exponential. See Fig. 1.13(b) and (c).

(fig.)

(fig..)

Fig. 1.13

While dealing with two-dimensional motion convert the problem into two one-dimensional motions. Separate v_x and v_y similarly a_x and a_y. Treat the motion in x- and y-directions.

Projectile

A freely falling body having constant horizontal velocity may be termed as a projectile. In general, in one direction the motion be accelerated and in another direction the motion is uniform, then such a motion is called "projectile motion". Fig. 1.14 shows acceleration in y-direction and uniform velocity in x-direction. Such bodies follow parabolic path.

(fig.)

Fig. 1.14 Projectile Motion

Oblique Projectile Motion

Assume a projectile is fixed at an angle θ with horizontal, with a velocity u from point O as shown in Fig. 1.15. Resolve velocity along x and y-axis. Along y-axis g acts then maximum height attained.

(fig.)

Fig. 1.15 Oblique Projectile Motion

$$h_{max} = \frac{u^2 \sin^2 \theta}{2g}$$

Time of flight T $= \frac{2\ u \sin \theta}{g}$

Horizontal range $R = \frac{u^2 \sin 2\theta}{g}$

Note that the range will be same if projected at complement angles, i.e. θ and $(90 - \theta)$ with same velocity.

Maximum range = $R_{max} = \frac{u^2}{g}$

when $\theta = 45°$

Trajectory $y = x \tan\theta - \frac{gx^2}{2u^2 \cos^2\theta}$

or $y = x \tan\theta \left[1 - \frac{x}{R}\right]$ Path is parabolic.

Instantaneous velocity = $|v|$

$$= \sqrt{u_x^2 + u_y^2} + \sqrt{u_x^2 + (u_y - gt)^2}$$

$$= \sqrt{u^2 + g^2t^2 - 2ugt \sin\theta}$$

$$\tan\beta = \frac{u_y - gt}{u_x}$$

Range and Time of Flight along an Inclined Plane

Consider an inclined plane of inclination α. Let a projectile be fixed at an angle θ with the horizontal or at an angle $(\theta - \alpha)$ with respect to incline plane as shown in Fig. 1.16.

The time of flight $T' = \frac{2u \sin(\theta - \alpha)}{g \cos\alpha}$

Range $R' = \frac{2u^2 \sin(\theta - \alpha) \cos\theta}{g \cos^2\alpha}$

$$R = \frac{u^2}{g \cos^2\alpha} [\sin(2\theta - \alpha) - \sin\alpha]$$

(fig.)

Fig. 1.16 Projectile motion along an incline

Range R' along the inclined is maximum if $2\theta - \alpha = \frac{\pi}{2}$ or $\theta - \alpha = \frac{\pi}{2} - \theta$. That is, R' is maximum when the direction of projection bisects the angle that the inclined plane makes with Oy' and $R'_{max} = \frac{u^2}{g \cos^2\alpha} \cdot [1 - \sin\alpha]$

N.B. In projectile motion along the plane acceleration acts along x and y axis both.

CIRCULAR MOTION

Concepts and Equations

Circular motion may be divided into two types (i) motion in a horizontal circle (ii) motion in a vertical circle. In vertical circle acceleration due to gravity plays a role and hence speed at every point is different. We will deal with them separately.

Horizontal Circular Motion

Acceleration is continuously required to change the direction even though if the speed is constant. Therefore, equations of motion used in translation cannot be applied. We define new variables and equations to describe motion.

(i) Angular Displacement (θ)

Change in angular position (initial to final) is "called angular displacement" as shown in Fig. 1.16. Unit is radian.

(ii) Angular Velocity (ω)

Time rate of change of angular displacement is called "angular velocity" $\omega = \frac{d\theta}{dt}$. Average angular velocity = $\frac{2\pi}{T}$ where T is time period of revolution. If a revolution is not completed then $\omega_{av} = \frac{\text{angular displacement}}{\text{time taken}}$. Unit is rad s^{-1}.

(fig.)

Fig. 1.16

(iii) Angular Acceleration (α)

Time rate of change of angular velocity is called "angular acceleration", $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$. If angular acceleration is uniform, then $\omega = \omega_0 + \alpha t$; $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ and $\omega^2 - \omega_0^2 = 2\alpha\theta$ can be applied. Here ω_0 is initial angular velocity, θ is angular displacement and t is time. If $\alpha = 0$, then particle moves in uniform circular motion.

Note that speed v remains unchanged.

Relation between v and ω; a and a_t

$v = r\omega$ where r is radius of the circle. Note from Fig. 1.17 that velocity is tangential.

(fig.)

Fig. 1.17 Tangential or linear velocity

$\alpha_t = r\alpha$ where α_t is tangential acceleration. Fig. 1.18 shows there are two accelerations, one along the radius called radial or centripetal or normal acceleration α_t. The other is tangential or along the tangent. Thus, net acceleration a_{net} is

(fig.)

Fig. 1.18 Net acceleration ilustration

$a_{net} = \sqrt{a_r^2 + a_t^2}$ and $\tan\beta = \frac{a_r}{a_t}$ with respect to tangential acceleration.

Centripetal Force

$F_e = \frac{mv^2}{r} = mr\,\omega^2$. It is a pseudo force and acts forwards the centre.

Centrifugal Force

The inertial reaction required to take into account the acceleration of frame of reference is called centrifugal force and is equal to $-mr\,\omega^2$.

Motion in a Vertical Circle

When a body moves in a vertical circle then at the highest point $\frac{mv^2}{r} \geq mg$ or $v_{min} = \sqrt{rg}$ at the highest point. Minimum velocity, v_{min} at point P can be determined using the face that the body has come down by $AL = AO + OL = r + r\cos\theta$ as shown in Fig. 1.19.

(fig.)

Fig. 1.19 Velocity at any point in a vertical circle

Thus $v^2_{min\ p} = v^2_{min\ top} + 2\ gh$

$= rg + 2g(r + r\cos\theta)$

or $v_{min,p} = \sqrt{3rg + 2rg\cos\theta}$

v_{min} at the lowest point or bottom is obtained by using $\theta = 0$, $\cos\theta = 1$.

or $v_{min,bottom} = \sqrt{5rg}$

Tension, if the string is used or normal reaction at any point P is obtained as T or N = $\frac{mv^2}{r} + mg\cos\theta$ (See Fig 1.20) where v is velocity.

(fig.)

Fig. 1.20 Tension at any point P in a vertical circle.

NEWTON'S LAWS OF MOTION

Concepts and Equations

Force is pull or push which generates or tends to generate motion, in a body at rest, stops or tend sot stop a body in motion , increases or decreases the magnitude of velocity of the moving body, changes or tends the shape of the body.

Newton's First Law of Motion

If a body is at rest will remain at rest and a body in uniform motion will remain in the state of uniform motion unless it is compelled by some external force to change its state.

Inertia

It is the inherent property of the body with which it cannot change by itself state of rest or of uniform motion unless acted upon by an external force. Hence, Newton's first law of motion may also be called law of inertia.

N.B. that the term external force has been used in first law. It means there would be internal force also.

Internal Force

If the force applying agent lies in inside the system, force is internal. Internal force cannot provide motion. For example, if you are sitting in a car and you push the car, car does not move. If you come out of the car and apply the same force, car moves. When you were inside the car, the force applying agent was inside the car, hence, the force was internal and car did not move. When the force applying agent (you) had moved outside, the car moved.

The straight line along which force acts is called "line of action of the force".

In order to accelerate or decelerate a body an unbalanced force is required.

A system of bodies on which no external force acts is called a "closed system". For example, two bodies moving towards each other due to their mutual electrostatic or gravitational force.

When many forces act on a body at the same point, they are called "concurrent forces". The system of concurrent forces may be:

(a) Collinear, that is acting along the same straight line.

(b) Coplanar, that is in the same plane.

(c) Generally directed, but not in the same plane.

Mass

In Newtonian mechanics mass is considered to be a measure of inertia of a body and is considered independent of its velocity. It is scalar quantity. Unit → kg (SI system).

Momentum

The total quantity of motion contained in a body is called "momentum". It is a vector quantity. Unit kg ms^{-1} (SI) $\vec{p} = m\vec{v}$.

If two different masses have same momentum, then the lighter one has more kinetic energy (also more velocity).

Newton's Second Law of Motion

The time rate of change of momentum is directly proportional to force (external) applied on it and the change in momentum occurs in the direction of force.

$$\vec{F} \propto \frac{dp}{dt}, \quad \text{or} \quad \vec{F} = \frac{d\vec{p}}{dt} = \frac{md\vec{v}}{dt} = m\vec{a}$$

Newton considered mass to be constant. Unit of Force is Newton (N) or kg Wt (kilogram weight) or kg f(kilogram force) 1 kg Wt = 1 kg of = 9.8 N.

If mass is varying and velocity constant $F = v\frac{dm}{dt}$ if both mass and velocity vary, $F = \frac{dmdv}{dt}$.

Impulse

Product of force and time for which it acts is called "impulse".

$$F = \frac{dp}{dt} \quad \text{or} \quad F, dt = dp$$

i.e. impulse = change in momentum. $F_{av} \cdot t = \Delta p$ is called "impulse momentum theorem"

Newton's Third Law of Motion

To every action there is an equal and opposite reaction, i.e., $\vec{F}_{AB} = -\vec{F}_{BA}$. Moreover, action and reaction act on different bodies. According to third law forces in nature occur in pairs. Single isolated force is not possible.

N.B. In certain cases of electrostatics and in springs Newton's third law fails.

Law of Conservation of Momentum

If no external force acts then the total momentum of the system is conserved.

$$\vec{F} = \frac{d\vec{p}}{dt} = 0 \quad \text{or} \quad \vec{p} = \text{constant.}$$

Equilibrium

Translatory Equilibrium

When several forces act on a body such that resultant force is zero, i.e., $\Sigma F = 0$, the body is said to be in "translatory equilibrium". $\Sigma F = 0$, implies $\Sigma F_x = \Sigma F_x = \Sigma F_z = 0$. It means the body is in the state of rest (static equilibrium) or in the uniform motion (dynamic equilibrium).

If the force is conservative then
$F = \frac{du}{dr} = 0$ means potential energy
u = maximum, minimum or constant.

Stable Equilibrium

If on slight displacement from equilibrium position, body has the tendency to regain its original position. In such cases Centre of Mass (COM) rises on slight displacement. Note that PE is minimum $\left(\frac{d^2u}{dr^2} = +ve\right)$ in stable equilibrium.

Unstable Equilibrium

If on slight displacement from equilibrium position the body moves in the direction of displacement, the equilibrium is known to be unstable. The COM goes down on slight displacement. PE is maximum and $\frac{d^2u}{dr^2} = -ve$ for unstable equilibrium.

Neutral Equilibrium

If the body remains at the dispatched position after a slight displacement then such an equilibrium is neutral. The COM does not change and PE is constant but not zero.

Fig. 1.21 illustrates all the types of equilibrium: stable, unstable and neutral.

(fig.)

Fig. 1.21 Types of equilibrium explanations

Strings

String is considered to be massless unless stated and hence, tension remains constant throughout the string.

String is assumed to be inextensible unless stated and hence, acceleration of any number of masses connected to it is always equal or same. If the pulley is massless and smooth, and string is also massless then tension at each point (or two sides of string) is constant as shown in Fig. 1.22.

(fig.)

Fig. 1.22 Tension in string for a light and smooth pulley

If the string changes tension changes as illustrated in Fig. 1.23.

(fig.)

Fig. 1.23 Tension in different strings

T_1, T_2 and T_3 in the Fig. 1.23 are different as string changes. In Fig. 1.23 $T_3 = 2T_1$

$T_2 = 2(g - a)$

(fig.)

Fig. 1.24

If forces are equal and opposite on a massless string as shown in Fig. 1.24 then tension T is equal to either of the two forces, i.e. T = F.

The maximum tension which a string can bear is called its “breaking strength”. If the string has mass tension is different at each point as illustrated in Fig. 1.25.

(fig.)

Fig. 1.25 Illustration of tension in a stringrod having mass

Mass per unit length $\lambda = \frac{M}{l}$. We have to find tension at P, Mass of (l – x) part is $\frac{M(l - x)}{l}$.

$$\text{Tension at P} = \frac{F}{M}\left(\frac{M(l - x)}{l}\right) = \frac{F(l - x)}{l}.$$

Springs

Springs are assumed massless unless stated. Restoring force is same every where, i.e. $F = -kx$.

Spring can be stretched or compressed. Stretch or compression is taken positive.

Restoring force is linear as is clear from $F = -kx$, k is called "spring constant or force constant".

$k \propto \frac{1}{l}$ (k also depends upon radius, length and material used).

In series $\frac{1}{k_{effective}} = \frac{1}{k_1} + \frac{1}{k_2} + \dots$

In parallel $k_{effective} = k_1 + k_2 + \dots$

If masses m_1 and m_2 connected to a spring as shown in Fig. 1.26 are oscillating or both masses move then find reduced mass μ; $\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$.

(fig.)

Fig. 1.26

If the spring has mass m, then $\frac{m_s}{3}$ is used to produce extension.

Pseudo Forces

The hypothetical forces added while dealing with problems associated with non-inertial or accelerated frame of reference, so that Newton's laws may be applied are called "pseudo forces or inertial forces". If a frame of reference is moving with an acceleration a_0, then force on a particle of mass m is ma_0. In the force equation force $-ma_0$ will be added to make the frame of reference inertial.

Friction

If we try to slide a body over a surface the motion is resisted by the bonding between the body and the surface. This resistance is represented by a single force called "friction". The friction is parallel to the surface and opposite to the direction to intended motion. Remember static friction is a self adjusting force. If a body is at rest and not being pulled, force of friction of is zero. If a pulling force is applied and the body does not move, friction still acts and is called "static friction". The maximum value of static friction is called "limiting friction". See Fig. 1.27. if we apply the force beyond limiting friction, the body begins to move and friction slightly decreases called "kinetic friction".

(fig.)

Fig. 1.27 Friction illustration

Limiting Friction

$F_\mu = \mu$, N where N is normal reaction $\mu_s = \tan \mu$ where μ is the angle of limiting friction.

N.B. $\mu_s > \mu_k > \mu_R$ where μ_s stands for coefficient of static friction, μ_k stands for coefficient of kinetic friction and μ_R stands for rolling friction.

Friction is independent of surface area of contact. However, it depends upon the nature of material of the surfaces in contact, their roughness, smoothness, inclination, Normally friction between too smooth bodies is more. If the bodies are made extra smooth by polishing the bonding force of cohesion or adhesion increases resulting in cold welding.

In practice $0 < \mu$ 1 but $\mu > 1$ is observed. For example; $\mu_s = 1$ for glass/glass, and, $\mu_s = 1.6$ for Cu – Cu. Friction is a non-conservative force.

If force is applied and still the body is at rest then the force of the friction is equal to force applied.

Equation of motion for centre of mass (COM)

$$m \frac{dv_{COM}}{dt} = \Sigma F$$

WORK, POWER AND ENERGY

Concepts and Equations

Work

The work is said to be done when a particle is displaced by the action of a force. It is a scalar quantity. Unit of work is Joule (SI) and CGS unit is erg. Practical unit of work (particularly in electric consumption) is kWh. is 1Wh = 3.6×10^6 J and 1J = 10^7 ergs. Sometime eV is also used. 1 eV = 1.6×10^{-19} J. In problems of heat 1 calorie = 4.186 J

$$dW = \vec{F} \cdot \vec{d}s$$

$W = \vec{F} \cdot \vec{s} = Fs \cos \theta$ if force is constant throughout.

$W = \int F \cdot ds$ if force is variable.

Work is positive or negative depending on the value of θ. For acute angles $\cos \theta$ is positive and hence, work is positive. For obtuse angle $\cos \theta$ may be negative making work negative. Positive work is parallel to displacement and negative work is opposite is displacement.

Work done in lifting a body up (against gravity) is positive and work done by the force of gravity (vertically downward motion) is negative.

No work will be done if the body is in static or dynamic equilibrium, i.e., W = 0 if $\Sigma F = 0$.

No work will be done if displacement is zero or force is perpendicular to the displacement. Thus, work done by centripetal force and work done by moving charged particle in a magnetic field is zero i.e., $F = q (\vec{v} \times \vec{B})$ will do no work. Work done depends upon the frame of reference. If frame of reference is changed displacement may very and hence work done could be different in different frame of reference.

In a conservative field work done is path independent.

$$W = \Delta PE = \int F \cdot ds.$$

In a force versus displacement curve, work done is area under the graph. The algebraic sum of the area is to be found out as illustrated in Fig .1.28.

(fig.)

Fig. 1.28

$W = \int_{v_1}^{v_2} PdV$ Area under Pressure (P) and Volume (V) curve is work done.

$W = \Delta KE$, i.e., work = change in KE. This is also called "work energy theorem".

For positive work $KE_{final} > KE_{initial}$. Work energy theorem is valid for all types of forces (internal or external, conservative or nonconservative).

In case of a spring $W = \frac{1}{2} kx^2$ where x is extension or compression in the spring.

$W = \frac{1}{2}$ stress × strain × volume in elastic bodies.

Since work is independent of time. We define, time rate of doing work is Power.

$$P = \frac{dW}{dt} = \frac{d}{dt}(\vec{F} \cdot d\vec{s})$$

$$= \vec{F} \cdot \frac{d\vec{s}}{dt} = \vec{F} \cdot \vec{v}$$

Power is a scalar quantity. Its SI unit is Watt (W) or J/s. Practical unit of power is HP or bhp (british horse power) 1 bhp = 746 W = 550 ft – lb/s.

$W = \int P \cdot dt$ or area under P – t graph.

N.B. KE can never be negative while P'E can be both negative or positive. Potential energy is defined only for conservative force. It does not exist for nonconservative forces.

Elastic PE = $\frac{1}{2}kx^2$ and is taken positive in all cases.

Electric PE = $\frac{q_1q_2}{4\pi\epsilon_0 r}$ may be negative or positive.

Gravitational PE = $-\frac{GM_1M_2}{r}$ may be negative or positive.

Mechanical energy = KE + PE is conserved if internal forces are conservative and no work is done by nonconservative forces. It some of the internal forces are nonconservative mechanical energy of the system is not conserved.

Total energy = KE + PE + internal energy.

Internal energy is directly related to temperature. Larger the internal energy, higher is the temperature of the body.

Thermal energy is related to random motion of molecules while internal energy is related to motion as well as their configuration or arrangement.

$E = mc^2$ is mass energy relationship.

Quantization of Energy

Planck has shown that the radiations emitted by a black are quantized. Quantum nature of energy is confirmed in atomic and subatomic world. Even light energy is quantized.

CONSERVATIONOF MOMENTUM

Concepts and Equations

Momentum

Momentum or linear momentum $\vec{p} = m\vec{v}$. Unit $kgms^{-1}$ or Ns. $p = [MLT^{-1}]$.

Linear momentum depends upon the frame of reference. For instance linear momentum of a body at rest in a moving train is zero with respect to a person sitting in the train while it is not zero with respect to ground.

Momentum is direction dependent. Thus two bodies having equal speed but different direction will have different momentum.

(a) $p \propto v$ if m is constant, i.e. for particles of equal mass, momentum will be maximum for a particle having largest velocity.

(b) $p \propto m$ if v is same, i.e. the heaviest particle will have maximum momentum if the particles have same velocity.

(c) If p = constant then $v \propto \frac{1}{m}$. , i.e., for particles having same momentum the lightest particle will have maximum velocity and hence maximum K.E.

$$\frac{p^2}{2m} = KE$$

$\therefore \quad p = \sqrt{2(KE)m}$

(d) If p = const. KE $\propto \frac{1}{m}$ i.e. lightest particle will have maximum KE if the particles have equal momentum. See Fig. 1.29.

(e) If KE of some particles is equal then the heaviest one will have maximum momentum i.e., $p \propto \sqrt{m}$. See Fig. 1.29

(f) If m is constant then $p \propto \sqrt{(KE)}$. See Fig. 1.29(c).

Since $\vec{F} = \frac{d\vec{p}}{dt}$, the slope of p-t curve will yield force and the area under F-t curve will give the change in momentum or impulse.

N.B. $p = \frac{h}{\lambda}$ for a particle wave and $p = \frac{E}{c}$ for photons.

(fig.)

(fig..)

Fig. 1.29

Law of Conservation of Momentum

If $F_{ext} = 0$ then Σp_i = constant, i.e. linear momentum of various particles may change but their vector sum remains unchanged.

Law of conservation of momentum is independent of frame of reference though momentum depends on the frame of reference.

Law of conservation of momentum is equivalent to Newton's third law of motion. $\vec{p}_1 + \vec{p}_2$ = const. or $\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0$ or $F_1 + F_2 = 0$ or $F_2 = -F_1$, i.e. for every action there is an equal and opposite reaction.

Law of conservation of momentum is universal, i.e. it can be applied to microscopic as well as macroscopic particle. It holds good even in atomic and nuclear physics where classical physics fails.

Collision or Impact

It is an isolated even in which a strong force acts for a short interval. The motion of colliding particles (at least one of them must) change abruptly. During collision particles may or may not come in physical contact. For example, in collision between two balls, balls come in physical contact but in collision of charged particles like α-particle scattering there is no physical contact.

In collision, we consider the situation just before and just after impact. The duration of collision is negligibly small as composed to the time for which event is observed. During collision internal forces act on the colliding particles.

If the motion of the colliding particles before and after impact remains in the same straight line, the collision is said to be "direct or head on or one-dimensional collision".

N.B. In one-dimensional collision velocity of COM of the colliding particles is in the same straight line.

If two particles after collision do not maintain the same line of motion, the collision is said to be "oblique". If in an oblique collision particles before and after collision remain in the same plane then collision is said to be "two-dimensional", otherwise, it is "three-dimensional".

Effect of external forces like friction, gravity is not considered in collision as duration of collision is very small. Average impulsive force responsible for collision is much greater than the external force acting on the system.

If charge on the interacting particles remains unchanged during collision, the process is termed as scattering. If the charge changes then reaction is the name given to such a process. However total charge remains conserved.

The impulsive force acting during collision is internal and hence, the total momentum of the system remains conserved.

If in a collision KE before and after collision are equal collision is said to be "elastic". Collision between atomic and subatomic particles may be elastic.

If in a collision, colliding particle stick together or more with a common velocity after collision then such a collision is perfectly "inelastic". For example, a bullet embedded in a wooden block after collision.

Most of the collision in our world (macroscopic) are imperfect or partially inelastic. For such collision we define coefficient of restitution (e).

$$e = \frac{\text{Velocity of Separation}}{\text{Velocity of approach}} = \frac{v_2 - v_1}{u_1 - u_2}$$

N.B. e = 0 mass collision is perfectly inelastic and for e = 1, collision is perfectly elastic and for 0 < e < 1 collision is partially inelastic.

One-dimensional collision (Elastic) $\mu_1 > \mu_2$

(fig.)

(fig..)

Fig. 1.30

Conserving momentum and KE, we can write

$$v_1 = \left(\frac{m_l - m_2}{m_1 + m_2}\right) u_1 + \frac{2m_2}{m_1 + m_2} u_2$$

$$v_2 = \left(\frac{m_l - m_2}{m_1 + m_2}\right) u_2 + \frac{2m_2}{m_1 + m_2} u_1$$

N.B.

1. If $m_1 = m_2$ $v_1 = u_2$ and $v_2 = u_1$ i.e. velocities after collision interchange.
2. If target particle is at rest, i.e., $u_2 = 0$ and $m_1 = m_2$, $v_1 = 0$, $v_2 = u_1$.
3. If target is massive $m_2 >> m_1$ and is at rest $v_1 = u_1$ and $v_2 = 0$.
4. If projectile is massive $v_1 = u_1$ and $v_2 = 2u_1 - u_2$. If target is at rest then $v_2 = 2u_1$.

Partially Inelastic Collision

To solve problem conserve momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ and exploit $e = \frac{v_2 - v_1}{u_1 - u_2}$.

Remember in oblique collision coefficient of restitution e be employed only along common normal.

Perfectly Inelastic Collision

(fig.)

(fig..)

Fig. 1.31 Just after collision

Oblique Collision

Assuming two dimensional collision. Conserve momentum in x and y direction separately. If collision is elastic KE is also conserved. Remember KE is scalar.

Therefore, do not take the components along x and y direction. If two particles have equal mass and collision is oblique elastic then equal masses fly off at right angle to one another.

Motion of Two Masses Connected by a Spring

Assume spring is massless. The spring is compressed or stretched by x, so that m_1 is displaced by x_1 and m_2 by x_2 then $F_{ext} = 0$.

$\therefore \quad \vec{p}_1 + \vec{p}_2 = 0$ or $p_2 = p_1$

$|\vec{p}_1| = |\vec{p}_2|$

$\therefore \quad \frac{KE_1}{KE_2} = \frac{m_2}{m_1}$, i.e. lighter block moves faster or has more KE

or CoM is at rest.

Therefore

$$m_1x_1 + m_2x_2 = 0$$

and $x = x_2 + x_2$

(fig.)

Fig. 1.32

Note KE of blocks is not constant.

Centre of Mass (COM)

COM of a body is a point where the whole mass of the body may be assumed to be concentrated for dealing with its translatory motion. For a discrete system of particles coordinates of COM may be determined.

as $\quad r_{COM} = \frac{m_1r_1+m_2r_2+\ldots m_nr_n}{m_1+m_2+m_3+\ldots+m_n} = \frac{\Sigma m_ir_i}{\Sigma m_i}$

or $\quad x_{COM} = \frac{\Sigma m_ix_i}{\Sigma m_i}$

$$y_{COM} = \frac{\Sigma m_iy_i}{\Sigma m_i}$$

and $\quad z_{COM} = \frac{\Sigma m_iz_i}{\Sigma m_i}$

If the mass is uniformly distributed then take a mass element dm at positive $\vec{r}$ as a point mass and replace the summation by integration. i.e.,

$r_{COM} = \frac{1}{m}\int r dm$ so that

$$x_{COM} = \frac{1}{m}\int x dm;$$

$y_{COM} = \frac{1}{m}\int y dm$ etc.

There may or may not be any mass present at the COM. It may be within or outside the body.

For symmetrical bodies having uniform distribution of mass, it coincides with centre of symmetry or geometrical centre.

If the COM of the parts of a system is known, combined COM may be obtained by treating mass of the parts concentrated at their respective centre of masses.

If COM is the origin of a coordinate system the sum of moments of masses of the system about origin vanishes (zero).

COM and centre of gravity in uniform gravitational field coincide but may be different for varying gravitational fields. For example, COM of a mountain may not coincide with its centre of gravity.

ROTATIONAL MOTION

Concepts and Equations

In Rotation Motion

We consider pure rotation and rolling. Rolling is basically rotational motion + linear motion.

Moment of Inertia (MOI)

Moment of inertia is a tensor. It plays the same role in rotational motion as mass in the linear motion. Moment of inertia $I = \Sigma m_i r_i^2$.

$I = \int r^2\, dm$ if mass is uniformly distributed; $I = Mk^2$ where M is total mass of the body and k is radius of gyration.

Radius of Gyration (k)

It is the root mean square perpendicular distance of the body from axis of rotation.

$$k = \sqrt{\frac{m_1 r_1^2 + m_1 r_2^2 + \ldots + m_n r_n^2}{m_1 + m_2 + \ldots + m_n}} = \frac{1}{M}\int r^2\, dm$$

Given below are MOI of the bodies about an axis passing through their COM (centre of mass) and perpendicular to the plane of the body.

MOI of a Ring $I_{ring} = MR^2$

MOI of a disc (solid) $I_{disc} = \frac{MR^2}{2}$

MOI of an annular disc (Fig 1.33)

$I_{annular\ disc} = \frac{M}{2}(R_1^2 + R_2^2)$

(fig.)

Fig. 1.33 Annular disc

MOI of a solid cylinder $I_{cylinder} = \frac{MR^2}{2}$

MOI of a hollow cylinder = MR^2 (if shell type, i.e, extremely thin walls).

$= \frac{M}{2}(R_1^2 + R_2^2)$ (Fig. 1.34)

(fig.)

Fig. 1.34 Hollow cylinder

MOI of a spherical shell = $\frac{2}{3}MR^2$

MOI of a solid sphere = $\frac{2}{5}MR^2$

MOI of a hollow = $\frac{2}{5}M(R_1^2 + R_2^2)$

MOI of a rod (cylindrical) = $\frac{Ml^2}{12}$ (Fig. 1.35)

(fig.)

Fig. 1.35 MOI of a rod

MOI of a rod (rectangular)

$$= \frac{M(l^2 + b^2)}{12}$$

MOI of a Lamina (rectangular)

$$= \frac{M}{12}(F + b^2)$$

MOI of a parallelepiped = $\frac{M}{12}(F + b^2)$ (Fig. 136)

(fig.)

Fig. 1.36 Rectangular lamina)

MOI of a elliptical disc = $\frac{M}{4}(a_2 + b_2)$ (Fig. 1.37)

(fig.)

Fig. 1.37 Elliptical disc

MOI of a cone (Right circular cone)

$$= \frac{3}{10} MR^2 \text{ (Fig. 1.38)}$$

(fig.)

Fig. 1.38 MOI of prism

MOI of a prism or equilibrium triangle

$$= \frac{M\,l^2}{6}$$

MOI of a triangle lamina (about base)

$$= \frac{M\,b^2}{6} \text{ (See Fig. 1.39)}$$

MOI of a triangle lamina about perpendicular $I_p = \frac{M\,p^2}{6}$

MOI of a triangle lamina about hypotenuse $I_h = \frac{mb^2p^2}{6(p^2 + b^2)}$

(fig.)

Fig. 1.39 MOI of a triangular lamina

MOI of a cone about XOX

$$= \frac{3}{5} M \left(\frac{R^2}{4} + h^2 \right) \text{ (Fig. 1.40)}$$

(fig.)

Fig. 1.40 MOI of a cone

MOI of a rod about one end = $\frac{M\,l^2}{3}$ (Fig. 1.41)

(fig.)

Fig. 1.41 MOI of a rod

Parallel Axis Theorem

If MOI about an axis passing through COM of a body is known, the MOI of the body about an axis parallel to the axis passing through COM and at a distance x from it as illustrated in Fig. 1.42 is

(fig.)

Fig. 1.42 Parallel axis theorem illustration

$I = I_{COM} + Mx^2$ where I_{COM} is the MOI about an axis passing through their COM.

Perpendicular Axis Theorem

It can be applied only to plane lamina bodies. If x-and y-axes chosen in the plane of the body and z-axis be perpendicular to this plane, these being mutually perpendicular, then

$I_z = I_x + I_y$ where I_x and I_y are MOI about x-axis and y-axis respectively.

(fig.)

Fig. 1.43 Perpendicular axis theorem illustration

Angular velocity (instantaneous)

$$\omega = \frac{d\theta}{dt}$$

Angular acceleration $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

Linear velocity $\upsilon = r\omega$; tangential acceleration $a_t = r\alpha$

$$\omega = \omega_0 + \alpha t;\ \theta = \omega_0 t + \frac{1}{2}\alpha t^2;$$

$$\omega^2 = \omega_0^2 + 2\,\alpha\,\theta$$

Torque (τ) $\vec{\tau} = \vec{r} \times \vec{F} = I\,\alpha$

$|\vec{\tau}|$ = Force $\times$ perpendicular from the axis of rotation.

$\vec{\tau} = \frac{dL}{dt}$ where L is angular momentum.

N.B. Torque is moment of a force about a point. Though dimensions of torque are same as that of energy but it is not energy. Its unit is N – m. Dimensional formula is $[ML^2\,T^{-2}]$.

If line of action of a force passes through its COM then such a force will not form torque.

Angular Momentum is moment of momentum (linear) about a point, i.e.,

$\vec{L} = \vec{r} \times \vec{p}$.

$\vec{L} = I\,\omega$; $|\vec{L}| = \Sigma p \times$ (Perpendicular distance form axis of rotation).

N.B. If external torque is zero then angular momentum is conserved.

Dimensional formula of L = $[ML^2\,T^{-1}]$. Its unit is kg $m^2\,s^{-1}$ and is same as that of Planck's constant h.

Angular impulse

$$J = \int_{t_1}^{t_2} \tau \cdot dt = \Delta L = L_2 - L_1$$

Rotational kinetic energy = $\frac{1}{2} I\,\omega^2$.

Note if a body only rotates about a fixed axis then it posses only rotational K.E. If, however, a body rolls then it possess both rotational KE and linear KE, i.e.

Total KE = Rotational KE + Linear KE

$= \frac{1}{2} I\,\omega^2 + \frac{1}{2} m v^2$

Work done $W = \int \vec{\tau} \cdot d\vec{\theta}$;

Rotational Power $P_{rot} = \vec{\tau} \cdot \vec{\omega}$.

Acceleration of a body rolling down an incline plane: In Fig. 1.44.

(fig.)

Fig. 1.44 Acceleration of a body rolling down an incline

$$a = \frac{g \sin\theta}{1 + \frac{I}{MR^2}} = \frac{g \sin\theta}{1 + \frac{k^2}{R^2}}$$

Velocity on reaching ground

$$v = a \cdot t = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$$

Time taken to reach the ground

$$t = \sqrt{\frac{2l\left(1 + \frac{k^2}{R^2}\right)}{g \sin\theta}}$$

For a system to be in rotational equilibrium $\Sigma\vec{\tau} = 0$

For a system to be in linear equilibrium $\Sigma F = 0$

For total equilibrium (Rotational + Linear) $\Sigma\tau = 0$, $\Sigma F = 0$.

Combined Motion
(Rotation + Translation)

(fig.)

Fig. 1.45 Velocities of different points of a wheel

$$\vec{a}_{COM} = \frac{F_{ext}}{M}$$

and $\alpha = \dfrac{\tau^{ext}_{COM}}{I_{COM}}$

These equations together with initial conditions completely define the motion. τ_{COM}^{ext} is external torque about COM.

$\upsilon_{COM} = r\omega$

$\upsilon_{bott} = 0; \upsilon_{top} = 2\,\upsilon_{COM}$

$= 2r\omega$ (See Fig. 1.44(a))

In Fig. 1.44 (b) in pure rolling

$$\upsilon_p = \omega\sqrt{r^2 + x^2}$$

Pure rolling $\upsilon_{COM} = r\omega$, the wheel completes 1 rotation and covers a distance $= 2\pi r$.

Rolling with forward Slipping

If the wheel moves a distance $> 2\pi r$ in one complete rotation then $\upsilon_{COM} > r\omega$ and motion is termed as "rolling with forward slipping".

Rolling with backward Slipping

If the wheel moves a distance $> 2\pi r$ in one complete rotation then $\upsilon_{COM} < r\omega$ and motion is known as rolling with backward slipping.

Angular Momentum of a body in Combined Rotation and Translation

$L = L_{COM} + M(\vec{r}_0 \times \vec{\upsilon}_0)$ where $M(\vec{r}_0 \times \vec{\upsilon}_0)$ is assumed to be the angular momentum as if mass is concentrated at COM and translating with υ_0. In an accelerating wheel force of friction acts in the direction of motion. So that frictional torque acts in a direction to oppose the accelerating torque.

If the wheel is rolling with forward slipping then force of friction acts in a direction opposite to the motion of the wheel until pure rolling begins.

Three-dimensional rotation is understood from gyrostal.

A spinning top shows (i) spinning (ii) precession (iii) nutation or wobbling

Hipparchus in 135 BC found that due to precession of earth ($T_{precession}$ = 27,725 yrs) a change in the direction of the line of equinoxes occurs and phenomenon is called "precession of equinoxes".

(fig.)

Fig. 1.46 Friction in accelerating wheel

Table 1.3 Equivalence between Rotational and Translation Motion

Linear Motion	Rotational Motion
Displacement = x	Angular displacement = θ
Linear velocity $\upsilon = \frac{dx}{dt}$	Angular velocity $\omega = \frac{d\theta}{dt}$
Acceleration $a = \frac{d\upsilon}{dt}$ mass = m	Angular acceleration $\alpha = \frac{d\omega}{dt}$ MOI = I
Linear momentum $p = m\upsilon$	Angular momentum $L = I\omega$

Force F = ma	Torque $\tau = I\,\alpha$
Impulse $I = \int F dt = \Delta p$	Rotational impulse $J = \int \tau\, dt$
Work $W = \int \vec{F} \cdot d\vec{x}$	Work $W = \int \vec{\tau} \cdot d\vec{\theta} = \Delta L$
$KE = \frac{1}{2} m\upsilon^2$	Rotational KE = $1/2\, I\, \omega^2$
Power $P = \vec{F} \cdot \vec{\upsilon}$	Rotational Power = $\vec{\tau} \cdot \vec{\omega}$

GRAVITATION

Concepts and Equation

Gravitational Force is first Natural Force

The modern science came to notice. It started from planetary motion and then took the shape as we know today.

Newton's Law of Gravitation

Newton is 1665 formulated $F \propto m_1 m_2$

$$F \propto \frac{1}{r^2} \text{ or } F = \frac{Gm_1m_2}{r^2}$$

Where $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} and is called "universal gravitational constant". The value of G was first measured by Cavendish in 1736. The value of G measured for small distances (r < 200 m) is less by about 1% and perhaps gives an indication of a fifth natural force. Note, gravitational field is independent of the nature of medium between the masses.

(fig.)

Fig. 1.47 Gravitational force between two masses

Gravitational Field Intensity

Gravitational force per unit mass is called "gravitational filed intensity". Gravitational field intensity of earth is 'g'.

$$E_g = \frac{F}{m} = \frac{GM}{r^2}$$

$$\text{and } g = \frac{GM_e}{R_e^2}$$

Gravitational field intensity due to a ring at any point on the axial line as illustrated in Fig. 1.48 is

$E_g = \frac{GMx}{(x^2 + R^2)^{3/2}}$. E_g is maximum if x

$= R/\sqrt{2}$.

(fig.)

Fig. 1.48 Gravitational field intensity at a point on the axial line

Gravitational field due to disc at any point on the axial line.

$$E_g = \frac{2GM}{R^2}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

$= \frac{2GM}{R^2}[1 - \cos\theta]$ in terms of angle θ.

Gravitational filed intensity due to a shell

$$E_{g\ inside} = 0, E_{g\ surface} = \frac{GM}{R^2}$$

$$E_{g\ out} = \frac{GM}{x^2}\ x > R$$

See Fig. 1.49

(fig.)

Fig. 1.49 Gravitational field intensity due to a shell

Gravitational field intensity due to a solid sphere

$$E_{g\ inside} = \frac{GMx}{R^2}\ x < R$$

$$E_{g\ surface} = \frac{GM}{R^2}\ x = R$$

$$E_{g\ outside} = \frac{GM}{R^2}\ x > R$$

See Fig. 1.50

(fig.)

Fig. 1.50 Gravitational field intensity due to a solid sphere

Gravitational Potential (V_g)

The amount of work done to bring a unit mass form infinity to that point under the influence of gravitational filed of given mass M without changing the velocity

$$V_g = \frac{-GM}{r} = \int_{\infty}^{r} E_g \cdot dx$$

Gravitational potential due to a ring at any point on the axial line.

$$V_g = \frac{-GM}{\sqrt{x^2 + R^2}}$$

Gravitational potential due to a shell

$$V_{inside} = V_{surface} = \frac{-GM}{R}\ x \le R$$

$$V_{outside} = \frac{-GM}{x}\ x > R$$

See Fig. 1.51

(fig.)

Fig. 1.51 Gravitational potential due to a solid sphere

$$V_{inside} = \frac{-GM}{2R^3}[3R^2 - x^2]\ x < R$$

$$V_{surface} = \frac{-GM}{R}\ x = R$$

$$V_{outside} = \frac{-GM}{x}\ x > R$$

See Fig. 1.52

(fig.)

Fig. 1.52 Gravitational potential due to a solid sphere

Gravitational Potential Energy

It is the amount of work done to bring a mass m from infinity to that point under the influence of gravitational field of a given mass M without changing the velocity $u_g = \frac{-GMm}{r}$. Note, that $u_g = mV_g$.

$$\text{Work done } W = \Delta u_g$$

Variation of ‘g’ due to height

$$g' = g\left(1 - \frac{2h}{R}\right)$$

if $h << R$

$$g' = \frac{g}{(1 + h/R)^2} \text{ if h is comparable to R}$$

Variation of ‘g’ due to depth

$$g' = g\left(1 - \frac{x}{R}\right) \text{ where x is depth.}$$

$g' = 0$ if $x = R$ i.e. at the centre of the earth.

Variation of ‘g’ with rotation of the earth lattitude

$g' = g\left(1 - \frac{R\omega^2}{g}\cos^2\lambda\right)$, i.e., g is maximum at the poles where $\lambda = 90°$) and minimum at the equator (where $\lambda = 0$).

Orbital Velocity

$\upsilon_0 = \sqrt{\frac{GM}{r}}$ Orbital velocity υ_0 is the velocity with which a planet or a satellite moves in its orbit of radius r.

Escape Velocity

Escape velocity is the minimum velocity given to a body so that it escapes (from the surface of the earth/planet) from its gravitational field, $\upsilon_e = \sqrt{\frac{2GM}{r}}$.

N.B. $\upsilon_e = \sqrt{2}\,\upsilon_e$.

$$\text{Time period } T = \frac{2\pi r}{\upsilon_0}$$

or $$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$KE = \frac{1}{2} m\upsilon_0^2 = \frac{GMm}{2r}$$

$$\therefore \quad PE = \frac{-GMm}{r}$$

Total energy or Binding energy

$$= KE + PE = \frac{-GMm}{r}$$

Kepler’s Law of Planetary Motion

First Law

The plants revolve around the sun in the elliptical orbits with sun at one of the focus as illustrated in Fig. 1.53.

(fig.)

Fig. 1.53 Kepler’s Ist Law illustration

Second Law

A line from the sun to the planet sweeps equal area in equal intervals of time as shown in Fig. 1.53. This law is based on conservation of angular momentum. From Kepler's 2nd law one can easily derive.

$$\frac{\upsilon_1}{\upsilon_2} = \frac{r_1}{r_2} = \frac{\upsilon_{perihelion}}{\upsilon_{aphelion}} = \frac{r_{aphelion}}{r_{perihelion}}$$

(fig.)

Fig. 1.53 Kepler's 2nd Law illustration

i.e., when the planet is closer o the sun it moves faster.

Closest distance of a planet from the sun is called "perihelion distance". Farthest distance of the planet from the sun is called "aphelion distance".

(fig.)

Fig. 1.54

Perihelion and aphelion distance illustration

Third Law

The square of the time period of a planet is proportional to the cube of the semimajor axis, i.e., $T^2 \propto r^3$. If e is the eccentricity of an elliptical orbit then.

(fig.)

Fig. 1.54 Kepler's 3rd law illustration

$$\frac{r_{Aphelion}}{r_{Perihelion}} = \frac{1+e}{1-e}, r_{Aphelion} + r_{Perihelion}$$

= 2r r being semimajor axis.

Schwazrschild Radius

$R_S = \frac{2GM}{c^2}$ where c is speed of light with radius R_S.

Event Horizon

The surface of the sphere with radius, R, surrounding a black hole is called "event horizon". Since, light cannot escape from with in this sphere, we cannot see events occurring inside.

Weightlessness in a Satellite

$\frac{GMm}{r^2} - N = \left(\frac{GM}{r^2}\right) m$ or N = 0 where N is normal contact force exerted by the surface. That is in a satellite surface does not exert any force on the body. Hence, apparent weight of the body is zero.

III. FLUID MECHANICS

Concepts and Equations

The density of a homogeneous substance is defined as mass per unit volume. Density is characteristic of a particular type of material and independent of the total quantity of material in the sample

$$P \equiv \frac{m}{V}$$

The SI units of density are kilograms per cubic meter

$1 g/cm^3 = 1000 kg/m^3$

Pressure is defined as the magnitude of the normal force per unit area acting on a surface. Pressure is a scalar quantity

$$P \equiv \frac{F}{A}$$

The SI units of pressure are newtons per square meter, or pascal (Pa).

$$1 Pa \equiv 1 N/m^2$$

Atmospheric pressure is often expressed in other units: atmospheres, mm of mercury (Torr),;or pounds per square inch.

1 atm = 1.013×10^5 Pa

1 Torr = 133.3 Pa

1 lb / in^2 = 6895 Pa

$P = P_0 + pgh$

$P_0 = 1.013 \times 10^5$ Pa = 1 atm

The absolute pressure, P, at a depth h below the surface of a liquid, which is open to the atmosphere, is greater than atmospheric pressure, p_o, by an amount which depends on the depth below the surface. The quantity $P\text{-}P_o = pgh$ is called the "gauge pressure" and P is the "-absolute pressure". The pressure has the same value at all points at a given depth and does not depend on the shape of the container

Fig.

Fig. 1.54

Pascal's law states that a change in the pressure applied to an enclosed fluid (liquid or gas) is transmitted undiminished to every point within the fluid and to the walls of the container.

Archimedes's principle states that when an object is partially or fully immersed in a fluid, the fluid exerts an upward buoyant force on the object. The magnitude of the buoyant force equals the weight of the fluid displaced by the object. The weight of the displaced fluid depends on the density, ρ_{fluid}' and volume of the displaced fluid, V.

$$B = \rho_{fluid} gv$$

For a floating object, the fraction of the volume that is below the fluid surface is equal to the ratio of the density of the object to that of the fluid. In Equation below V_{fluid} is the volume of the displaced fluid, and is therefore the volume of the object that is submerged.

$$\frac{Vfluid}{Vobj} = \frac{\rho_{obj}}{\rho_{fluid}}$$

The ideal fluid model is based on the following four assumptions:

1. nonviscous — internal friction between adjacent fluid layers is negligible.
2. incompressible — the density of the fluid is constant throughout the fluid.
3. steady flow — the velocity at each point in the fluid remains constant.
4. Irrotational (nonturbulent) — there are no eddy currents within the fluid; each element of the fluid has zero angular momentum about its center of mass

The equation of continuity for fluids

states that the flow rate (product of area and speed of flow) of an incompressible fluid (p = constant) is constant at every point along a pipe.

$$A_1 v_1 = A_2 v_2 = \text{constant}$$

The equation of continuity for fluids states that the flow rate (product of area and speed of flow) of an incompressible fluid (p = constant) is constant at every point along a pipe

$$A_1 v_1 = A_2 v_2 = \text{constant}$$

Bernoulli's equation states that the sum of pressure, kinetic energy per unit volume, and potential energy per unit volume remains constant along a streamline of an ideal fluid. The equation is a statement of the law of conservation of mechanical energy as applied to an ideal fluid,

$$P + \frac{1}{2}\rho v^2 + \rho gy = \text{constant}$$

Any thing that can flow is called a "fluid". Therefore, liquids and gases fall in this category. A perfect liquid is compressible and its shearing stress not maintainable.

Streamlined, Steady State Flow. By steady state or stationary flow we mean that at any place in a fluid, the velocity never changes.

Equation of Continuity Volume leaving per second = volume entering per second

or $A_1v_1 = A_2v_2$

Where A and A_1 and A_2 are Barea of cross-section of a pipe at enterance and leaving points and v_1 and v_2 are velocities at the respective points as shown in Fig. 1.55 (a) Fig 1.55 (b) shows variation of velocity with area of cross-sections.

(fig.)

Fig 1-55 (a) Equation of continuity illustration

(fig.)

Fig 1-55 (b) Velocity vs area of cross-section

Bernoulli's Theorem

It states that total energy of a (flowing) liquid is constant. That is,

KE + PE + Pressure head energy

= Constant

$$\frac{1}{2}mv^2 + mgh + P\Delta V = \text{constant}$$

or $$\frac{1}{2}pv^2 + pgh + P = \text{constant}$$

Torricelll's Theorem According to this theorem velocity of efflux.

$v_{efflux}\sqrt{2g(H-h)}$. See Fig. 1.56

(fig.)

fig 1.56 Velocity of efflux from an open vessel.

Velocity of efflux from a open vessel having Pressure P inside (See Fig 1.57)

fig

Fig. 1.57 Velocity of efflux from a closed vessel

$P = P_{atm} + \frac{1}{2}pv^2$ where P_{atm} is atmospheric pressure.

or $$v = \sqrt{\frac{2(P - P_{atm})}{p}}$$

Dynamic Lift or Magnus Effect: When a ball is spinning in a fluid as shown in Fig. 1-58 the resultant velocity at the top (above the ball) increases and resultant velocity below the ball decreases. If ν_1 and ν_2, are velocities of liquid and spinning ball respectively. The ball experiences an upward thrust. Such a phenomenon is called "dynamic lift or Magnus effect".

fig

Fig 1.58 Dynamic lift illustration.

Venturimeter

$$P_1 - P_2 = H\rho g$$

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$$

or $$\frac{P_1 - P_2}{\rho g} = \frac{v_2^2 - v_1^2}{2g}$$

or $$\frac{H\rho g}{\rho g} = \frac{V^2}{2g}\left[\frac{1}{a^2} - \frac{1}{A^2}\right]$$

$$V = aA\sqrt{\frac{2Hg}{\text{[illegible]}}}$$

and gives volume flowing per second.

fig

Fig 1.59 Venturimeter

Surface Tension: The property of the liquid with which the surface behaves as a stretched membrane and can support small objects placed on its surface.

Surface tension = Surface energy

Surface energy is defined as work done due to surface; tension per unit area. Dimensional formula of surface tension. $[MT^{-2}]$ Force per unit length. Consider two molecules of a liquid X on the surface and Y inside as .illustrated in Fig.1.60 is completely balanced due to forces exerted by other molecules of the liquid. X has unbalanced forces. The Figure: clearly demontrates why the surface of a liquid behaves as a stretched membrane. The molecular force between the molecules of a liquid is cohesive force. Due to surface tension a liquid would try to acquire minimum surface area maximum volume. Therefore, the drops acquire spherical' shape

Fig.

Fig. 1.60 Surface tension illustration

Excess pressure in a drop/bubble

$\Delta P = \frac{2T}{r}$ if the bubble has one surface like air bubble. Where T is surface tension and r is radius.

If a bubble has two surfaces, like soap bubble, then excess pressure is $\Delta p = \frac{4T}{r}$. See Fig 1.61

Fig.

fig..

Fig. 1.61

Angle of contact is the angle between the tangent to the liquid vapour interface and liquid solid interface. In other words, it is the angle between the tangent to the meniscus (at the point of contact) and wall of the container. As illustrated pig in Fig. 1.62 convex upward meniscus makes obtuse angle of contact and concave upward meniscus makes acute angle

Fig

Fig. 1.62

"Liquids like water, alcohol, ether, CCl_4 (Carbon tetrachloride) xylene, glycerine and acetic acid have angle of contact zero or nearly zero with glass. Meniscus may be concave upward or convex upward.

Table 1.4: Comparison of concave upward and vex upward meniscus

Concave meniscus	Convex upward Meniscus
1. Angle of contact is acute.	Angle of contact is obtuse
2. A dhesive force between the liquid molecules and molecules of glass (wall of the container) is greater than cohesive force between liquid molecules.	Cohesive force between liquid molecules is greater than adhesive force between the molecules of liqui8d and wall of the container.
3. The liquid wets the walls of the container	The liquid does not wet the walls of the container.

Hg has angle of contact 138° with the glass and angle of contact of water with chromium is 160°.

Ascent of liquid in a capillary tube

$h = \frac{2T\cos\theta}{r\rho g}$ if meniscus is not taken into account.

$h = \frac{2T\cos\theta}{r\rho g} - \frac{r}{3}$ if meniscus is also taken into account.

$h = \frac{2T}{r\rho g}$ if $\theta = 0$

Note that: If angle of contact θ is acute or meniscus is concave liquid rises in the capillary. If the angle of contact is obtuse or meniscus is convex upward then liquid dips as cos θ will be negative as shown in Fig. 1.63

Fig

Fig1.63 (a)

Fig

Fig. 1.63 (b)

If the liquid rises in a capillary and capillary is of insufficient height then the excess liquid will collect in the form of a drop at the top as shown in Fig. 1.64 but does not fall. That is no overflow will take place. If overflow would have taken place liquid would have risen again. Thus, a perpetual motion would have begun and such a motion is disallowed

Fig

Fig. 1.64

In Jaeger's method surface tension is estimated by $T= \frac{r}{2}$ (Ha-hp)g where r is radius of the capillary, is density of Xylol and p is the density of liquid under investigation, h is the depth of the capillary below the surface and H is the difference in two levels of the U tube. Fig. 1.65 illustrates Jaeger's method.

Fig

Fig. 1.65 Jaeger method illustration

Quinke's method: If a big drop is placed on a clean glass plate then angle of contact θ is determined by (see Fig. 1.66)

Fig

Fig. 1.66 Angle of contact measurement using Quinke's method

$\cos 0 = 1 - \frac{h^2}{H^2}$ The method is applicable for liquids making obtuse angle of contact or which do not wet the wall of the container.

Velocity of a simple harmonic wave on the surface of a liquid $v\sqrt{\frac{\lambda g}{2\pi}}$ It is valid only if the amplitude of circular vibration is very small as compared to the wavelength π . If the amplitude is large then $\nu = \sqrt{\frac{\lambda}{2\pi}\left(g + \frac{4\pi^2 T}{\rho\lambda^2}\right)}$

Energy E required to split a big drop of radius R into n small drops each of radius r

$$E = 4\pi r^2 n^{2/3} \left[n^{\frac{1}{3}} - 1\right]$$

$$T = 4\pi R^2 \left[n^{\frac{1}{3}} - 1\right] T$$

Where $R = n^{1/3} r$

Same amount of energy will be released when n drops each of radius r coalesce to form a big drop of radius R

Viscosity: The property of a fluid to oppose relative motion between its layers is called “viscosity”. This property can be observed when the flow is steady or the liquid moves with a constant velocity. The flow may be called “laminar”. The opposition is due to intermolecular forces (cohesive force) fig 1.67 shows the velocity of the layers decreases in a direction perpendicular to the flow

fig

Fig 1.67 Viscosity shows velocity gradient

Therefore, shearing stress $\frac{F}{A} \alpha$ velocity gradient $\frac{dv}{dy}$

(Rate of change of strain).

or $F = -\eta A \frac{dv}{dy}$ where η is coefficient of viscosity.

Dimensions of η $[ML^{-1}T^{-1}]$ SI unit Poiseuille (Pl) and COS unit is Poise

1 Pl = 10 poise.

Since the coefficient of viscosity is the ratio of shearing stress to the rate of change of strain, it may be regarded as "transient or fugitive rigidity". Maxwell regarded viscosity as the limiting case of elastic solid.

Note: Viscosity of liquids is greater than that of gases. For example, viscosity of water is 0.01 poise while that of air is 200 μ poise.

The motion profile of a liquid in a capillary is parabolic as shown in Fig.1.68

Fig

Fig. 1.68 Motion profile of a liquid in a capillary (Note parabolic profile)

$$v = \frac{p}{4\eta L}(r^2 - y^2)$$

Velocity of flow is maximum when y=0 (along the axis of tube) and is $v_{max} = \frac{pr^2}{4\eta L}$

r is radius of the tube, L is length of tube and p is pressure difference.

Velocity of flow is zero at the walls when y = r. v = 0. Critical velocity. The velocity at which steady or laminar flow changes to turbulant or eddy flow is called critical velocity.

Reynolds number R = $\frac{p v_c D}{\eta}$
Reynolds number R is dimensionless.

or $v_c \frac{R\eta}{\rho D}$

It has been found if R < 2000, flow is steady and if R> 3000 flow is turbulent. For water R < 2000 corresponds to v< 20cms at 20°C.

or $= \frac{\rho v_c D}{\eta} = \frac{\frac{1}{2} p v^2}{\frac{\eta v}{r}} = \frac{\text{inertial force}}{\text{viscous drag}}$ D is diameter of the tube.

Kinematic viscosity is $\frac{\eta}{\rho}$. Its unit is stokes.

Stoke's formula: The viscous force opposing the motion of a sphere is F = 6π ηr ν when a sphere travels through a fluid illustrated in Fig. 1.69(a). Velocity u is called terminal

Fig

Fig 1.69

Terminal velocity v_r - $\frac{2r^2(\rho - o)g}{9\eta}$

where ρ is density of sphere or drop and σ is density of fluid (medium).

N.B. $v_r \alpha r^2$ and $v_r \alpha$ density ρ

Poiseuitle's equation: The amount of liquid flowing per second, through a tube of radius r is given by

$$\frac{dV}{dt} = \frac{\pi P r^4}{8\eta l}$$

where $\frac{P}{l}$ is pressure gradient and $\frac{P}{l} = \frac{\rho g(h_1 - h_2}{l}$ See Fig. 1.70 (a). Fig 1.70 (b) shows how rate of flow varies with pressure head.

Fig

Fig. 1.70(a)

Fig

Fig. 1.70(b)

Variation of viscosity with temperature in liquids $Hg^{-1/2} = AC^{cg/T}$ where A and Care constants, ρ is density and T is temperature in Kelvin., i.e., viscosity of the liquids decreases with rise in temperature.

For gases $\eta = \eta_o a\ T^{1/2}$, i.e., in gases viscosity increases with rise in temperature

In gases, coefficient of viscosity $\eta = \frac{1}{3}\lambda p C$ where λ is mean free path, ρ is density and C is rms velocity of the gas. Searles apparatus is used to measure the viscosity of gases

Viscosities of two liquids can be compared using viscometer. Ostwald viscometer is quite common.

$$\frac{\eta_1}{\eta_2} = \frac{\rho_1 t_1}{\rho_2 t_2}$$

where ρ_1 and ρ_1 are densities and t_1 and t_2 are times the two liquids take to vacate the viscometer.

Density: $\rho = \frac{dm}{dV}$ units kg m^{-3} [SI] and g cm^{-3} [CGS].

Relative density or specific gravity

$$= \frac{\text{densityofasubstance}}{\text{densityofwaterat}4^oC}.$$

It is dimensionless and represents density of substance in magnitude of CGS units, since in CGS unit ρ_{water} =1g cm-3. Variation of density with temperature $\rho = \rho_o (1-\gamma\Delta\theta)$ if ρ_1 is density of mass m_1 and ρ_2 density of mass m_2 then density of the combination is

$$\rho = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}} = \frac{\sum m_1}{\sum \frac{m_1}{\rho_1}}$$

$$\rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} \text{ if } m_1 = m_2$$

$$\rho = \frac{\rho_1 + \rho_2}{2} \text{ if } V_1 = V_2$$

density increases with rise in pressure

$\rho = \rho_o \left(1 + \frac{\Delta P}{B}\right) = \frac{V_o}{V}\ \rho$

Pressure (P) $P = \frac{dF}{dS}$ P = Patm + pgh at a depth h below the surface of a liquid.

below the surface of a liquid.

$P - P_{atm} = \rho gh$ is called "gauge pressure or partial pressure". Dimensions of pressure is $[ML^{-1}\ T^{-2}]$

1 atm = 760 torr = 1.0 1 x I 0^5 Pa and 1 torr = 1 mm of Hg.

Pressure is a scalar quantity as its direction is always normal to the area. Pressure is independent of amount of liquid, shape of the container or cross-sectional area. It depends only on depth below the surface, nature of the liquid (ρ) and acceleration due to gravity. Barometers are used to measure atmospheric pressure. Fortin's barometer is most common. Manometers measure partial pressure. Pirani and ionization gauge is used to measure vacuum.

Pascal law If an external pressure is applied to an enclosed fluid it is transmitted undiminished to every position of the fluid and to the walls of the container. That is at any point $P = (P_{atm} + \Delta P)\ \rho\ gh$ if we apply external pressure ΔP then

Archimedes' principle and buoyancy: When a body is immersed in a fluid wholly or partly, its weight decreases equal to the weight of the fluid displaced by the body. The upward thrust is called buoyancy and acts vertically upward (opposite to the weight) of the body through the centre of gravity of the displaced fluid (called centre of buoyancy).

Upthrust $F_{up} = \ell A\ \sigma g$ where σ is the density of fluid and ℓA is the volume displaced. Note, the situation of Fig. 1.71 carefully. See how the reading of base balance and hanging spring balance varies. When the suspension is independent of vessel, the base balance reading increases by upthrust or F . When the base of the vessel also holds the rigid support from where block is hanged via spring balance, then reading of the base balance is b + mg.

Newtons laws can be applied

Fig. 1.71

N.B: b is weight of vessel + liquid and mg is the weight of block

Floatation A body will float if the weight of the body mg $\leq$ upthrust or buoyant force,

i.e., if $\rho_{body} \leq \rho_{liquid}$

See Fig. 1.72

if $\rho_{body} < \rho_{liquid}$ it floats

if $\rho_{body} = \rho_{liquid}$ it just floats.

Fig. 1.72

MCQs

I. UNITS, DIMENSIONS AND VECTORS

1. The fundamental unit of length in SI system is
   (a) metre
   (b) foot
   (c) mile
   (d) yard

2. The SI unit of capacitance is
   (a) Henry
   (b) Farad
   (c) Ohm
   (d) Lux

3. Which of the following is not SI base unit?
   (a) kilogram
   (b) Ampere
   (c) mole
   (d) Rutherford

4. Curie is the unit of
   (a) luminous intensity
   (b) radioactivity
   (c) amount of substance
   (d) electric current

5. Which of the following is not a unit of radioactivity?
   (a) Curie
   (b) Becquerel
   (c) Ruthford
   (d) Candela

6. The SI derived unit of magnetic flux is
   (a) Lumen
   (b) Tesla
   (c) Henry
   (d) Weber

7. Lux is the SI unit of
   (a) luminous flux
   (b) illuminance
   (c) magnetic flux
   (d) inductance

8. Candela is the SI base unit of
   (a) illuminance
   (b) luminous flux
   (c) luminous intensity
   (d) radiant intensity

9. Candela per square is the SI unit of
   (a) luminous flux
   (b) luminous intensity
   (c) luminance

10. The SI unit of magnetic flux density is
    (a) Tesla
    (b) Henry
    (c) Weber
    (d) Ohm

11. In CGS system, the unit of magnetic field intensity is
    (a) Oersted
    (b) Ampere per meter
    (c) Ampere
    (d) Farad

12. The SI unit of solid angle is
    (a) steradian
    (b) radian
    (c) degree
    (d) Joule

13. Which of the following is not a unit of plane angle?
    (a) degree (b) radian
    (c) Gradian (d) steradian

14. A dimensionless quantity is a quantity without an associated physical dimension. Which of the following is not a dimensionless quantity?
    (a) radian
    (b) π (pi)
    (c) decibel
    (d) force

15. Generally one housepower (HP) is equal to how many watts?
    (a) 546 W
    (b) 646 W
    (c) 746 W
    (d) 846 W

16. In SI system, volt per meter is the unit of
    (a) electric filed strength
    (b) magnetic field strength
    (c) magnetic flux density
    (d) magnetic flux

17. In SI system, Ohm is the unit of
    (a) electrical conductance
    (b) electrical resistance
    (c) inductance
    (d) capacitance

18. A light-year is a unit of
    (a) mass
    (b) time
    (c) temperature
    (d) length

19. Which of the following is not a unit of length?
    (a) micron
    (b) yard
    (c) inch
    (d) Kelvin

20. One micron is equivalent to
    (a)10^{-2} m (b) 10^{-4} m
    (c) 10^{-6} m (d) 10^{-8} m

21. The Pascal is not the SI derived unit of
    (a) pressure
    (b) stress
    (c) tensile strength
    (d) work

22. Which of the following does not have the same dimensions?
    (a) energy, work, heat
    (b) pressure, stress, young's modulus
    (c) voltage, electromotive force, potential difference
    (d) electric flux, electric field, electric dipole moment

23. The dimensions of force are
    (a) MLT^{-2}
    (b) ML^2T^{-2}
    (c) $M^2L^2T^{-2}$
    (d) ML^2T^{-1}

24. The dimensions of torque are
    (a) MLT^{-1}
    (b) $ML^2\ T^{-1}$
    (c) $ML^2\ T^{-2}$
    (d) $M^2L^2\ T^{-2}$

25. Out of the following pairs, which one does not have the same dimension?
    (a) force and weight
    (b) pressure and stress
    (c) energy and work
    (d) capacitance and resistance

26. Siemens is the SI unit of
    (a) electric resistance
    (b) electric conductance
    (c) electric capacitance
    (d) electric inductance

27. The dimensions of stress are
    (a) $M\ L^{-1}\ T^{-2}$ (b) $ML^{-1}\ T^{-2}$

(c) $M L^{-2} T^{-1}$ (d) $M L^{-2} T^{-2}$

28. The dimensions of electric resistance are
(a) $M L T^{-2} A$
(b) $M L^{-1} T^{-2} A^{-1}$
(c) $M L^{2} T^{-3} A^{-3}$
(d) $M L^{2} T^{-3} A^{-2}$

29. The SI unit of absorbed radiation dose of ionizing radiation is
(a) Radian
(b) Joule
(c) Watt
(d) Gray

30. The SI unit of catalytic activity is
(a) Katal
(b) Angstrom
(c) Sievert
(d) Gray

31. The SI unit of power is
(a) Henry
(b) Watt
(c) mil
(d) Sievert

32. The SI derived unit of dose equivalent is
(a) Gray (b) Katal
(c) Sievert (d) Henry

33. One electronvolt (ev) is a unit of energy equal to approximately
(a) 1.602×10^{-9} J
(b) 1.602×10^{-19} J
(c) 1.602×10^{-29} J
(d) 1.609×10^{-39} J

34. The dimension of gravitational constant G are
(a) $M^{-1} L^{-1} T^{-1}$
(b) $M^{-1} L^{-2} T^{-2}$
(c) $M^{-1} L^{-3} T^{-2}$
(d) $M^{-1} L^{-2} T^{-3}$

35. Natural units are physical units of measurement based only on universal physical constants. Which of the following is a natural unit?
(a) speed of light (c)
(b) electric charge (e)
(c) characteristic impedance of free space (z_0)
(d) all of the above

36. The dimensions of Coulomb constant (k) are
(a) $Q^{-1} M^{-2} L^{-2} T^{-2}$
(b) $Q^{-2} M L^{3} T^{-1}$
(c) $Q^{-1} ML^{2} T^{-2}$
(d) $Q^{-2} ML^{-3} T^{-2}$

37. What are the dimensions of electric constant ($\in_0$)? (It is also called perimttivity of free space)
(a) $Q^{2} M^{-2} L^{-2} T^{-2}$
(b) $Q^{-2} M^{-2} L^{-1} T^{-3}$
(c) $Q M^{-1} L^{-3} T^{-1}$
(d) $Q^{2} M^{-1} L^{-3} T^{-2}$

38. Which of the following is SI base unit for temperature?
(a) Celsuis
(b) Kelvin
(c) Fahrenheit
(d) Rankine

39. Debye (D) is a CGS unit of
(a) electric dipole moment
(b) density
(c) temperature
(d) mass

40. In SI units, a day consists of
(a) 56400 sec
(b) 66400 sec

(c) 76400 sec
(d) 86400 sec

41. The unit for indicating mass on an atomic or molecular scale is
(a) Dalton
(b) Neper
(c) Ampere
(d) Volt

42. The Coulomb (C) is the SI derived unit of
(a) electric current
(b) electric voltage
(c) electric charge
(d) electric field

43. The tonne (t) is a unit of mass equal to
(a) 10 kg
(b) 100 kg
(c) 1000 kg
(d) 10,000 kg

44. The SI unit for force is
(a) Volt
(b) Ampere
(c) Hertz
(d) Newton

45. The SI unit for frequency is
(a) hectre
(b) Volt
(c) Farad
(d) Hertz

46. The volt (V) is a SI derived unit of electromotive force, commonly called
(a) voltage
(b) current
(c) charge
(d) power

47. The smallest unit of mass; yoctogram (yg) equals
(a) 10^{-15} g
(b) 10^{-18} g
(c) 10^{-21} g
(d) 10^{-24} g

48. One mile equals to how many kilometers?
(a) 0.609 km
(b) 1.609 km
(c) 2.609 km
(d) 3.609 km

49. Which of the following SI units is not named after any physicist?
(a) Hertz
(b) Joule
(c) Volt
(d) Candela

50. One yard is equivalent to
(a) 1 foot (b) 2 feet
(c) 3 feet (d) 4 feet

51. When two vectors have opposite directions, we can say that they are
(a) parallel
(b) perpendicular
(c) antiparallel
(d) none of these

52. Which of the following is a scalar quantity?
(a) weight
(b) force
(c) velocity
(d) kinetic energy (K.E.)

53. Which vector can be used to locate the center of mass of a collection of particles?
(a) unit vector
(b) position vector
(c) distance vector
(d) none of the above

54. Which are the two basic properties of a vector?
(a) curvature and direction
(b) magnitude and direction

(c) magnitude and sign
(d) curvature and sign

55. Vectors are often split into two or more orthogonal components. What is true of these components?
(a) they are antiparallel
(b) they are the same curvature
(c) they are same magnitude
(d) they are perpendicular

56. Let i, j and k be unit vectors.
If a = 3i – j + 2k, what is the magnitude of the vector a?
(a) 4
(b) 12
(c) 14
(d) $\sqrt{14}$

57. Which vector gives the displacement from one point to another in space?
(a) unit vector
(b) position vector
(c) distance vector
(d) none of the above

58. Which of the following is a scalar quantity?
(a) work
(b) energy
(c) power
(d) all of the above

59. A plane flying 500 MPH due north has a tail wind of 45 MPH. The resultant velocity is
(a) 545 mph due south
(b) 455 mph due north
(c) 545 mph due north
(d) 455 mph due south

60. A man walks 3 miles north and then walks 4 miles east. The resultant displacement is
(a) 1 mile NE
(b) 7 miles NE
(c) 5 miles NE
(d) 5 miles SE

61. A man pushes against the wall with 50 N of force. The wall does not move. The resultant force is
(a) 50 N
(b) 100 N
(c) 75 N
(d) 0 N

62. The resultant magnitude of two vectors
(a) is always positive
(b) can never be zero
(c) can be negative, positive or zero
(d) is usually zero

63. Which of the following is not true?
(a) velocity can be negative
(b) velocity is a vector
(c) speed is a scalar
(d) speed can be negative

64. Poynting vector is closely related to
(a) power
(b) intensity of filed
(c) energy density
(d) none

65. What is the rates of 1 nanometer to 1 attometer?
(a) 10^6 (b) 10^7
(c) 10^8 (d) 10^9

66. Which of the following units is different from others?
(a) volt
(b) kilo-watt-hour
(c) watt-sec
(d) electron-volt

67. The dimensional formula for torque is identical to
(a) kinetic energy
(b) pressure energy
(c) momentum of force
(d) all of the above

68. If F = 8i – 2j and r = 6i + 8k, then F. r will be
    (a) 48 units
    (b) 32 units
    (c) 8 units
    (d) 6 units

69. If P = 2i + 3j – k and Q = 4i + 6j – 2j, then the angle between P and Q will be
    (a) 0°
    (b) 45°
    (c) 60°
    (d) 75°

70. Choose the only scalar?
    (a) kinetic energy
    (b) momentum
    (c) torque
    (d) angular momentum

71. The magnitude of resultant of two equal forces is equal to either of the force. What is the angle between the two forces?
    (a) 0°
    (b) 60°
    (c) 120°
    (d) 180°

72. If a body's momentum increases by 20%, the percentage increase in its K.E. will be
    (a) 30
    (b) 40
    (c) 44
    (d) 54

73. Which of the following physical quantity has different units as compared to others?
    (a) weight of a body
    (b) tension in string
    (c) buoyant force
    (d) electromotive force (e.m.f.)

74. What is the ratio of angular speed of minute hand and hour hand of a watch?
    (a) 1: 6
    (b) 6: 1
    (c) 1: 12
    (d) 12: 1

75. The moment of linear momentum is called
    (a) impulse
    (b) torque
    (c) couple
    (d) angular momentum

76. When net torque acting on a system is zero, which of the following will be constant?
    (a) force
    (b) linear momentum
    (c) angular momentum
    (d) linear impulse

77. What is the ratio of the inertial mass to gravitational mass?
    (a) 0.5 (b) 1
    (c) 2 (d) 3

78. The dimensional formula for velocity gradient is identical to that of
    (a) velocity
    (b) time-period
    (c) frequency
    (d) angular acceleration

79. The momentum of a body decreases by 20%, the percentage decrease in K.E. will be
    (a) 28% (b) 36%
    (c) 44% (d) 56%

80. Choose the physical quantity whose dimensions are different from others?
    (a) kinetic energy (K.E.)
    (b) pressure energy
    (c) moment of force
    (d) moment of momentum

81. ML^0T^{-2} is the dimensional formula of
    (a) moment of inertia
    (b) viscosity
    (c) surface tension
    (d) angular acceleration

82. ML^2T^0 is dimensional formula for
    (a) inertia
    (b) energy
    (c) moment of inertia
    (d) moment of momentum

ANSWERS

1. a	2. b	3. d	4. b
5. d	6. d	7. b	8. c
9. c	10. a	11. a	12. a
13. d	14. d	15. c	16. a
17. b	18. d	19. d	20. c
21. d	22. d	23. a	24. c
25. d	26. b	27. b	28. d
29. d	30. a	31. b	32. c
33. b	34. c	35. d	36. d
37. d	38. b	39. a	40. d
41. a	42. c	43. c	44. d
45. d	46. a	47. d	48. b
49. d	50. c	51. c	52. d
53. b	54. b	55. d	56. d
57. c	58. d	59. c	60. c
61. d	62. c	63. d	64. b
65. d	66. a	67. d	68. a
69. a	70. a	71. c	72. c
73. d	74. d	75. d	76. c
77. b	78. c	79. b	80. d
81. c	82. c		

II. NEWTON'S LAWS OF MOTION, GRAVITATION, WORK AND ENERGY

1. Initial velocity of a body moving with uniform acceleration of 5 m/s^2, is 10 m/s. What will be the distance covered in 10 sec?
    (a) 150 m
    (b) 250 m
    (c) 350 m
    (d) 450 m

2. Newton's first law of motion supplies the definition of
    (a) distance
    (b) velocity
    (c) acceleration
    (d) force

3. The action and reaction forces referred in Newton's third law of motion
    (a) must act upon the same body
    (b) must act upon different bodies
    (c) must be equal in magnitude but need not have the same line of action
    (d) need not to be equal in magnitude but must have the same line of action

4. If a person can thrown ball to a minimum height h (vertically up) then the maximum distance up to which he can throw the ball is
    (a) h
    (b) 2h
    (c) 3h
    (d) 4h

5. When a particle is thrown up, it will have
    (a) different speed at the same height during ascent and during descent
    (b) same speed at the same heights during ascent and during descent
    (c) same speed at different heights during ascent and during descent
    (d) different speed at different height during ascent and during descent

6. A carpet can be cleaned by beating. This is in accordance with Newton's
    (a) first law
    (b) second law
    (c) third law
    (d) none of the above

7. A force of 100 N acts on a body of 5 kg for 10 seconds. What will be the velocity of the body?
   (a) 2 m/s
   (b) 20 m/s
   (c) 200 m/s
   (d) 2000 m/s

8. Which law of motion explains the phenomenon of swimming?
   (a) first
   (b) second
   (c) third
   (d) none of these

9. What acceleration will a force of 5N produce in a mass of 5 kg?
   (a) 1 m/s^2
   (b) 25 m/s^2
   (c) 1 m/s
   (d) 25 m/s

10. A fielder caught a ball of 150 gm moving at a rate of 20 m/s. If the catching process was completed in 0.1 sec, the force of the blow exerted by ball on the hands was
    (a) 30,000 N
    (b) 30,000 N
    (c) 300 N
    (d) 30 N

11. When a body accelerates,
    (a) its direction always changes
    (b) its mass always
    (c) its velocity always changes
    (d) it falls towards the earth

12. A rope attached to a post in the ground is pulled horizontally with a force of 100 N. The pole pulls back with a force of
    (a) 25 N
    (b) 50 N
    (c) 100 N
    (d) 200 N

13. If a vehicle is to gain momentum, it must
    (a) lose weight
    (b) move slowly
    (c) lose inertia
    (d) accelerate

14. If the initial velocity, the distance travelled and the time elapsed are known, which equation would you use to find acceleration?
    (a) $v_f = v_i + at$
    (b) $v_f^2 - v_i^2 = 2as$
    (c) $s = v_i t + \frac{1}{2} at^2$
    (d) $s = vt$

15. Which of the following will not accelerate?
    (a) the moon in its orbit
    (b) a tennis ball rebounding from ground
    (c) a stone in free fall
    (d) a car in which the engine thrust is equal to the friction

16. A body is moving with a velocity of V_1 m/s and after to seconds, the velocity changes to V_2 m/s. The average acceleration of body will be
    (a) $\frac{V_1 - V_2}{t}$ m/s^2
    (b) $\frac{V_2 - V_1}{t}$ m/s^2
    (c) $\frac{V_1 - V_2}{t}$ m/s^2
    (d) $V_1 + V_2 =$ m/s^2

17. A car accelerates for 105 at 6 m/s^2. What is its final speed if its initial was 4 m/s?
    (a) 30 m/s
    (b) 60 m/s
    (c) 34 m/s
    (d) 64 m/s

18. A car is travelling due east at 40 miles/hr turns south and maintains the same speed. What is the change in velocity of car?
    (a) 40 miles/hr E – s
    (b) 40 $\sqrt{2}$ miles/hr E – s
    (c) 80 miles/hr E – s
    (d) none

19. A force of 6N acts on a body of mass 1 kg during which the body attains a velocity of 30 m/s. The time for which the force acts on a body is
    (a) 26 seconds
    (b) 5 seconds
    (c) 6 seconds
    (d) 2 seconds

20. Which of the following is not necessary for work to be done?
    (a) a constant speed
    (b) an applied force
    (c) a displacement
    (d) force component along the displacement

21. A body at rest may have
    (a) speed
    (b) momentum
    (c) energy
    (d) velocity

22. What is the weight of 10 kg block?
    (a) 10 N
    (b) 9.8 N
    (c) 98 N
    (d) 0.98 N

23. A fixed pulley is employed to
    (a) same work
    (b) change the direction of force
    (c) do more work with the same force but without using the pulling
    (d) have mechanical advantage greater than one

24. Which is the suitable method to decrease friction?
    (a) lubrication
    (b) polishing
    (c) ball and roller bearings
    (d) all of the above

25. The static friction is
    (a) always equal to dynamic friction
    (b) always less than dynamic friction
    (c) always greater than dynamic friction
    (d) sometimes greater and sometimes less than the dynamic friction

26. The force of friction that comes into action after the motion has started is known as
    (a) static friction
    (b) dynamic friction
    (c) friction only
    (d) limiting friction

27. A body in equilibrium may not have
    (a) velocity
    (b) momentum
    (c) acceleration
    (d) kinetic energy (K.E.)

28. A body is termed as perfectly elastic if
    (a) it can move freely
    (b) its surface is perfectly smooth
    (c) it is not affected by external force
    (d) it recovers its original shape when the deforming force is removed

29. If two bodies undergo a collision that is not perfectly elastic, then
    (a) K.E. is conserved but momentum is not
    (b) momentum is conserved but K.E. is not
    (c) neither K.E nor momentum is conserved
    (d) both K.E. and momentum are conserved

30. A negative acceleration does not necessarily imply
    (a) a decreasing speed
    (b) an increasing distance
    (c) an increasing speed
    (d) a decreasing distance

31. If the average velocity of an object is zero in some time interval, the displacement of the object for that interval will be
    (a) infinite
    (b) zero
    (c) increasing
    (d) decreasing

32. A car is moving at constant speed toward the east, on a free way. Its acceleration is
    (a) zero
    (b) positive
    (c) negative
    (d) infinite

33. The average velocity depends on the
    (a) displacement vector and not on the path covered
    (b) both displacement vector and on the path travelled
    (c) neither displacement vector nor the path traveleld
    (d) none of these

34. The magnitude of the instantaneous velocity is called the
    (a) displacement
    (b) speed
    (c) acceleration
    (d) length

35. The path of a projectile is a
    (a) triangle
    (b) circle
    (c) ellipse
    (d) parabola

36. The horizontal range (R) of a projectile is maximum when projectile's angle is
    (a) 90° (b) 75°
    (c) 60° (d) 45°

37. In the absence of non-zero net force, the center of mass of a body either remains at rest, or moves at a constant speed in a straight line. This is a statement of Newton's
    (a) first law of motion
    (b) second law of motion
    (c) third law of motion
    (d) none of the above

38. The acceleration of an object is proportional to the net force acting on it and inversely proportional to its mass. This is a statement Newton's
    (a) first law of motion
    (b) second law of motion
    (c) third law of motion
    (d) all of the above

39. The total force applied on a body is equal to the time derivative of a linear momentum of the body. This is Newton's
    (a) Ist law of motion
    (b) 2nd law of motion
    (c) 3rd law of motion
    (d) none of the above

40. Which law of motion is sometimes referred to as the action-reaction law?
    (a) 1st law of motion
    (b) 2nd law of motion
    (c) 3rd law of motion
    (d) none of the above

41. The three laws of motion were first publishes in 1687 by Sri Isaac Newton in his work
    (a) Method of Fluxions (b) Optics
    (c) Philosophie Naturalis Principia Mathematica
    (d) Arithmetic Uniersalis

42. According to which law of motion forces in nature occur in pairs? (single isolated force is not possible).
    (a) first law
    (b) second law
    (c) third law
    (d) none of the above

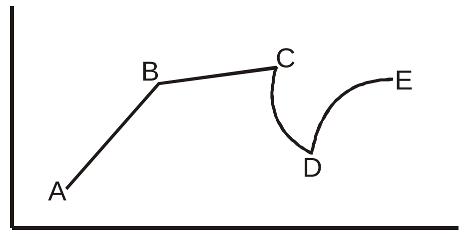
43. Newton's third law fails in certain cases of
    (a) electrostaties
    (b) electronics
    (c) springs
    (d) both a and b

44. If two different masses have same momentum, then the lighter one has more
    (a) K. E. (also more velocity)
    (b) potential energy (P.E.)
    (c) both K.E. and P.E.
    (d) none of the above

45. Two bodies of 1 kg and 4 kg are moving with equal K.Es. The ratio of the magnitudes of their linear momenta is
    (a) 1: 4
    (b) 8: 1
    (c) 1 : 16
    (d) 1 : 2

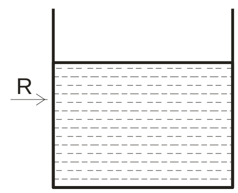
46. Which of the following is the magnitude of the gravitational force and is not the inherent property of body?
    (a) mass
    (b) weight
    (c) speed
    (d) length

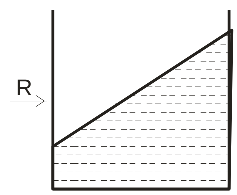
47. Fig. below shows displacement of a body going along the x-axis as a function of time. The force acting on the body is zero in the region.

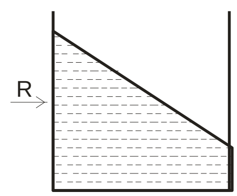


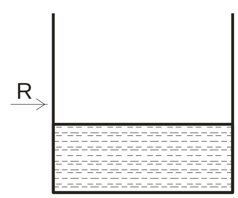
    (a) AB
    (b) BC
    (c) CD
    (d) DE

48. A jar containing water in placed in a train. The train accelerates from left to right. Which of the following figures shows the water level in a jar correctly?

(a) 

(b) 

(c) 

(d) 

49. Two objects A and B are thrown upwards simultaneously with the same speed. The mass of A is greater than that of B. Let air friction is constant
    (a) A will go higher than B
    (b) B will go higher than A
    (c) the two bodies will reach the same height
    (d) none of the above

50. Superman throws a 2400-N boulder at an adversary. What force must be apply to that stone to give it a horizontal acceleration of 12 m/s^2?
    (a) 244 N
    (b) 294 N
    (c) 2940 N
    (d) 29400 N

51. If the vector sum of forces on a body is not zero, the body
    (a) accelerates
    (b) decelerates
    (c) remains at rest
    (d) none of the above

52. Planets move around the sun due to
    (a) centrifugal force
    (b) centripetal force
    (c) gravitational pull between them
    (d) none of the above

53. Two bullets A and B have masses 1kg and 2kg respectively.
    (a) the k.E of B wil be twice that of A
    (b) the K.E. of A will be twice that of B
    (c) K.E. will be the same
    (d) none of the above

54. Both linear momentum and K.E. are conserved in
    (a) elastic collision
    (b) inelastic collision
    (c) both of the above
    (d) none of the above

55. A collision in which the two bodies stick together after the collision is
    (a) elastic collision
    (b) inelastic collision
    (c) both a and b
    (d) none of the above

56. The velocity of an object when projected from the earth in order to escape the earth's gravitational field is called the
    (a) terminal velocity
    (b) average velocity
    (c) instantaneous velocity
    (d) escape velocity

57. Which of the following is not an elastic collision?
    (a) a man jumps on a cart
    (b) a bullet embedded in a block
    (c) collection of two glass balls
    (d) none of the above

58. A shell explodes and many pieces fly off in different directions. The following is conserved
    (a) momentum
    (b) K.E.
    (c) both of the above
    (d) none of above

59. When the velocity of a body is doubled, which one is doubled too?
    (a) K.E.
    (b) acceleration
    (c) momentum
    (d) P.E.

60. K.E. of a body of mass m and momentum p is given by
    (a) p^2m
    (b) $m^2/2p$
    (c) mp
    (d) $p^2/2m$

61. If the K.E. of a body becomes four times of its initial value ,the new momentum will be
    (a) half
    (b) same
    (c) double
    (d) four times

62. Two bodies of mass m_A and m_B have equal K.E.. The ratio of their momentum is
    (a) $\sqrt{m_A} : \sqrt{m_B}$
    (b) $m_A : m_B$
    (c) $m_A^2 : m_B^2$
    (d) $m_B : m_A$

63. Which of the following quantities is zero about the centre of mass of a body?
    (a) mass
    (b) acceleration
    (c) moment
    (d) angular acceleration

64. Where should be the center of gravity of a body?
    (a) it must be within the body
    (b) it may be near but not essentially within the body
    (c) it must be outside the body
    (d) it changes its position after some time

65. If the earth stopped rotating, the weight of objects at either pole would
    (a) be greater
    (b) be less
    (c) vary with latitude
    (d) be the same before

66. The force of gravity between two objects does not depend upon
    (a) the constant of gravitation
    (b) the separation
    (c) the product of their masses
    (d) the sum of their masses (SHM)

67. The term "radius of gyration" relates to
    (a) moment of force
    (b) moment of inertia
    (c) law of gravitation
    (d) simple harmonic motion

68. The escape velocity
    (a) is independent of mass of the body
    (b) increases with the increase of mass of the body
    (c) decreases with the decrease of the body mass
    (d) depends upon the type of body used

69. What will happen to the force of gravity if the mass of one of the objects is tripled?
    (a) triple the original force of gravity
    (b) divide by 1/3rd of the original gravitational force
    (c) the gravitational force will remain the same
    (d) none of the above

70. If you have a hocky puck sliding along a table, it will eventually come to a stop. Which Newton's law this example illustrate?
    (a) Ist (b) 2nd
    (c) 3rd (d) none of these

71. Which Newton's law states the need to wear seatbelts?
    (a) Ist law
    (b) 2nd law
    (c) 3rd law
    (d) none of these

72. What is another name of the Newton's Ist law?
    (a) law of mass
    (b) law of inertia
    (c) law of velocity
    (d) law of acceleration

73. Which of the Newton's laws does this example illustrate. The blood in your head rushes to your feet, when riding on an elevator that is descending and abruptly stops?
(a) Ist (b) 2nd
(c) 3rd (d) none of these

74. German astronomer Kepler described the motion of planets in ———— laws
(a) 2 (b) 3
(c) 4 (d) 5

75. Which law of Kepler states that the planets revolve around the sun in then elliptical orbits with sun at one of the two foci?
(a) Ist (b) 2nd
(c) 3rd (d) none of these

76. Which of the Kepler's law is called the harmonic law?
(a) Ist (b) 2nd
(c) 3rd (d) none of these

77. A region of space from which nothing (not even light) can escape is called
(a) nova
(b) black hole
(c) white hole
(d) none of these

78. What is the period of geostationary satellite?
(a) 0 hour
(b) 12 hours
(c) 18 hours
(d) 24 hours

79. A wheel is 1m in diameter. When it makes 30 RPM the linear speed of a point on its circumference (in m/s) is
(a) π (b) $\pi/2$
(c) 30π (d) 60π

80. The angular speed of the second's hand of a watch in rad/see is
(a) π (b) $\pi/3$
(c) $\pi/2$ (d) $\pi/30$

ANSWERS

1. c	2. d	3. b	4. b
5. b	6. a	7. c	8. c
9. a	10. d	11. c	12. c
13. d	14. c	15. d	16. b
17. d	18. d	19. b	20. a
21. c	22. c	23. b	24. d
25. d	26. d	27. c	28. d
29. b	30. a	31. b	32. a
33. a	34. b	35. d	36. d
37. a	38. b	39. b	40. c
41. c	42. c	43. d	44. a
45. d	46. b	47. a	48. c
49. a	50. c	51. a	52. b
53. a	54. a	55. b	56. d
57. c	58. a	59. c	60. d
61. c	62. a	63. c	64. b
65. d	66. d	67. b	68. d
69. a	70. a	71. a	72. b
73. a	74. b	75. a	76. c
77. b	78. d	79. b	80. d

III. FLUID MECHANICS

1. Viscosity in fluids refers to
(a) the density of a fluid
(b) the compresibility of a fluid
(c) tangential force exerted on solid surface by the flowing fluid
(d) normal forces exerted on solid surface by the flowing fluid

2. Viscosity is a measure of the resistance of a fluid that is being deformed by either shear stress or tensile stress. Which of the following is more viscous?
(a) air (b) honey
(c) ketchup (d) water

3. Which law states that the pressure change in a confined incompressible fluid is transmitted equally in all directions throughout the fluid and to the walls of the container?
(a) Ohm's law
(b) Pascal's law
(c) Kirchoff's law
(d) Newton's law

4. All liquid surfaces tend to contract. This phenomenon is due to
   (a) viscosity
   (b) diffusion
   (c) density
   (d) surface tension

5. Which principle states that for an ideal fluid, an increase in the speed of fluid occurs stimultaneously with a decrease in fluid's pressure or P.E.?
   (a) Bernulli's principle
   (b) Archimedes' principle
   (c) both of the above
   (d) none of the above

6. The nib of fountain pen is split to convey ink down the nib by the phenomenon of
   (a) Adhesion
   (b) Cohesion
   (c) Osmosis
   (d) Capillary

7. The buoyancy depends upon the
   (a) depth to which the body is immersed
   (b) shape of the body
   (c) mass of body
   (d) mass of the liquid displaced

8. Which principle states that buoyant force on a submerged object is equal to the weight of the fluid displaced by the object
   (a) Bernoulli's principle
   (b) Archimedes' law
   (c) both of the above
   (d) none of the above

9. Bernoulli's equation includes a special case of
   (a) Archimede's law
   (b) Hooke's law
   (c) Torrcelli's law
   (d) Newton's law

10. The venturi-meter is an instrument used for measuring
    (a) the viscosity of a fluid
    (b) the specific gravity of a fluid
    (c) the flow speed of a liquid
    (d) the compressibility of a fluid

11. Turbulent flow of a fluid occurs when the Reynold number is a above about
    (a) 1000
    (b) 2000
    (c) 3000
    (d) 4000

12. Fluid flows a laminar for Reynold numbers up to
    (a) 500 (b) 1000
    (c) 2000 (d) 4000

13. For the Bernoulli's theorem to the applicable, the fluid flow should be
    (a) rotational and compressible
    (b) irrational and incompressible
    (c) irrotational (non-trubulent) and compressible
    (d) irrotational and incompressible

14. The ratio of the speed of an object moving through air to the speed of sound is called
    (a) Reynold number
    (b) Mach number
    (c) Avogadro's number
    (d) Feigenbaum number

15. Surface tension of a liquid may be defined as
    (a) heat energy per unit area
    (b) potential energy per unit area
    (c) surface energy per unit area
    (d) kinetic energy per unit area

16. Powder clings to the face due to
    (a) compression
    (b) capillary action
    (c) cohesion
    (d) adhesion

17. The ideal fluid model is based on four assumptions
    (a) nonviscous, incompressible, steady flow, non-turbulent
    (b) vsicous, compressible, steady flow, turbulent
    (c) nonviscous, compressible, steady flow, turbulent
    (d) nonviscous, incompressible, steady flow, turbulent

18. A beaker is full of water with an ice piece floating. The ice piece has a lead piece in it. When ice cube melts them
    (a) water overflows
    (b) level falls
    (c) level remains unchanged
    (d) none of these

19. The velocity at which laminar (steady) flow changes to turbulent (eddy) flow is called
    (a) terminal velocity
    (b) escape velocity
    (c) critical velocity
    (d) uniform velocity

20. The Magnus effect is equivalent to
    (a) Bernoulli's theroem
    (b) Archimedes' principle
    (c) Pascal's law
    (d) none of these

21. The viscosity of an ideal fluid is
    (a) infinity
    (b) unity
    (c) zero
    (d) 0.5

22. Out of the following, the maximum viscosity is of
    (a) oxygen
    (b) mercury
    (c) water
    (d) glycerine

23. The internal friction in a fluid is called
    (a) viscosity
    (b) surface tension
    (c) turbulence
    (d) none of these

24. The absolute viscosity of a fluid is primarily function of
    (a) density
    (b) temperature
    (c) pressure
    (d) velocity

25. Fluid mechanics is the study of how fluids move and the ———— on them.
    (a) energy
    (b) velocity
    (c) forces
    (d) position

26. Birds, planes and boats are streamlined to reduce
    (a) turbulence
    (b) thrust
    (c) lift
    (d) drag

27. Any substance that can flow is a
    (a) solid
    (b) gas
    (c) liquid
    (d) fluid

28. The design of an airfoil uses
    (a) Archimedes' principle
    (b) Bernoulli's principle
    (c) both of the above
    (d) none of the above

29. The force that moves a rocket or a plane forward is called
    (a) lift
    (b) drag
    (c) turbulence
    (d) thrust

30. Which principle is in effect when a ship displaces ocean water?
    (a) Burnoulli's
    (b) Ohm's
    (c) Pascal's
    (d) Archimedes

31. Which force pushes up a body in a fluid?
    (a) thrust
    (b) lift
    (c) buoyant
    (d) pressure

32. When air moves an airfoil, ———— is generated
    (a) thrust
    (b) lift
    (c) drag
    (d) turbulence

33. The study of the deformation and flow of matter, primary in the liquid state is called
    (a) Rheology
    (b) Geology
    (c) Physiology
    (d) Cosmology

34. In flow through a straight, smooth, pipe, the critical Reynolds number for transition to turbulence is generally taken to be
    (a) 1500
    (b) 2300
    (c) 4000
    (d) 10,000

35. Minor losses through valves, fittings, bends, etc., are modelled as proportional to
    (a) velocity head
    (b) static head
    (c) total head
    (d) pressure drop

36. With increase in temperature, the angle of contact of liquid
    (a) increases
    (b) decreases
    (c) becomes zero
    (d) first increases then decreases

37. A gas behaves as an ideal gas at
    (a) low pressure and high temperature
    (b) high pressure and low temperature
    (c) low pressure and low temperature
    (d) high pressure and high temperature

38. The colour of a star is an indication of its
    (a) size
    (b) weight
    (c) temperature
    (d) distance from the earth

39. What is measured by a Bolometer?
    (a) specific heat
    (b) thermal conductivity
    (c) heat radiation
    (d) e.m.f.

40. Newton's law of cooling is a special case of
    (a) Stefan's law
    (b) Wien's law
    (c) Kirchoff's law
    (d) Planck's law

41. What is the absorption power of perfect black body?
    (a) 1 (b) –1
    (c) 0 (d) infinity

42. The spectrum of a black body is
    (a) line
    (b) band
    (c) continuous
    (d) none of the above

ANSWERS

1. c	2. c	3. b	4. d
5. a	6. d	7. d	8. b
9. c	10. c	11. c	12. c
13. d	14. b	15. c	16. d
17. a	18. b	19. c	20. a
21. c	22. d	23. a	24. b
25. c	26. d	27. d	28. b
29. d	30. d	31. c	32. b
33. a	34. b	35. a	36. a
37. a	38. c	39. c	40. a
41. a	42. c		

IV. PERIODIC MOTION

1. Which law states in simple terms that stress (force per unit area) is directly proportional to strain (fractional deformation)?
   (a) Newton's
   (b) Hooke's
   (c) Kepler's
   (d) Ohm's

2. In simple harmonic motion (SHM), we have the conservation of
   (a) kinetic energy
   (b) potential energy
   (c) total energy
   (d) electrical energy

3. The angular frequency, time period and frequency in SHM does not depend upon
   (a) mass
   (b) force constant
   (c) amplitude
   (d) all of the above

4. The angular frequency, time period and frequency of a simple pendulum depends only on the
   (a) mass and amplitude
   (b) mass and acceleration (g)
   (c) amplitude and mass
   (d) length (L) and acceleration (g)

5. When the restoring force is proportional to the ———— from equilibrium, the oscillation is called simple harmonic motion (SHM)
   (a) displacement
   (b) velocity
   (c) time
   (d) frequency

6. A body that undergoes simple harmonic motion is called a harmonic
   (a) oscillator
   (b) amplifier
   (c) pendulum
   (d) none of the above

7. Which of the following physical systems are examples of simple harmonic oscillator?
   (a) mass on spring
   (b) mass on a pendulum
   (c) uniform circular motion
   (d) all of the above

8. The period of pendulum is determined by its
   (a) mass
   (b) amplitude
   (c) speed
   (d) length

9. SHM may be assumed as a projection of uniform circular motion along a
   (a) diagonal
   (b) hypotenuse
   (c) radius
   (d) diameter

10. A body experiences SHM with an amplitude. When this body is at its maximum displacement its phase is
   (a) $\pi/4$
   (b) $\pi/2$
   (c) π
   (d) 2π

11. The amplitude of a vibrating body placed in a resisting medium
    (a) increases exponentially with time
    (b) decreases exponentially with time
    (c) remain constant with time
    (d) none of the above

12. Two particles are executing SHM of same period. If the second particle starts form mean position T/2 later than the first the phase difference between the two particles at any instant of time is
    (a) $\frac{3\pi}{2}$ (b) $\frac{\pi}{4}$
    (c) $\frac{\pi}{2}$ (d) π

13. The total energy of a body executing SHM is directly proportional to
    (a) the amplitude
    (b) the square of the amplitude
    (c) square root of the amplitude
    (d) reciprocal of the amplitude

14. The circular motion of a particle with constant speed is
    (a) periodic and SHM
    (b) periodic and SHM
    (c) periodic but not SHM
    (d) neither periodic nor SHM

15. Two SHM are represented by $y_1 = 0.1 \sin (100\pi t + \pi/3)$ and $y_2 = 0.1 \cos \pi t$. The phase difference of the velocity of 1st with respect to 2nd is
    (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$
    (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{6}$

16. If E is the total energy of a particle experiencing SHM and A is the amplitude; the E and A are related as
    (a) $E \propto \frac{1}{A_2}$
    (b) $E \propto \frac{1}{A}$
    (c) $E \propto A$
    (d) $E \propto A^2$

17. The displacement of particle in SHM in one time period is
    (a) zero
    (b) a
    (c) 2a
    (d) 4a

18. The frequency of SHM is 100 Hz. Its time period is
    (a) 0.1 s
    (b) 0.01 s
    (c) 1s
    (d) 100 s

19. Which of the following is not essential for the free oscillation of a mass attached to a spring?
    (a) elasticity
    (b) gravity
    (c) inertia
    (d) restoring force

20. Which of the following quantities associated with SHM do not vary periodically?
    (a) velocity
    (b) displacement
    (c) acceleration
    (d) total energy

21. What is the number of degrees of freedom of an oscillating simple gravity pendulum?
    (a) 1 (b) 2
    (c) 3 (d) 4

22. The graph between restoring force and time in SHM is
    (a) straight line
    (b) parabola
    (c) sine curve
    (d) circle

23. How much radians is the phase difference between the velocity and displacement is SHM?
(a) π (b) $\frac{\pi}{2}$
(c) 2π (d) 0

24. What is the time period of a seconds pendulum?
(a) 1 sec (b) 2 sec
(c) 3 sec (d) 4 sec

25. The time period of the hour hand of a watch is
(a) 1 h (b) 6 h
(c) 12 h (d) 24 h

26. The curve between the acceleration and velocity of body in SHM is a(an)
(a) circle
(b) parabola
(c) ellipse
(d) triangle

27. Which of the following exhibits chaotic behaviour?
(a) double pendulum
(b) inverted pendulum
(c) pendulum
(d) none of the above

28. The potential energy of a simple pendulum at rest is 10 J and its mean kinetic energy is 5 J. Its total energy at any instant will be
(a) 5 J (b) 10 J
(c) 15 J (d) 20 J

29. The time period of a torsional pendulum is
(a) $T = \pi\sqrt{\frac{C}{I}}$ (b) $T = 2\pi\sqrt{\frac{C}{I}}$
(c) $T = 2\pi\sqrt{\frac{I}{C}}$ (d) $T = \pi\sqrt{\frac{I}{C}}$

30. Restoring force in the simple harmonic motion is
(a) centripetal
(b) frictional
(c) conservative
(d) nonconservative

31. The time period of the second's hand of a watch is
(a) 1 s (b) 1 min
(c) 1 h (d) 12 h

32. The mean kinetic energy of a harmonic oscillator with respect to position is
(a) $\frac{Ka^2}{2}$ (b) $\frac{Ka^2}{3}$
(c) $\frac{Ka^2}{4}$ (d) $\frac{Ka^2}{6}$

33. What is the maximum time period of a simple pendulum?
(a) 84.6 min
(b) 1 day
(c) 12 h
(d) 1 year

34. The differential equation representing SHM of a particle is $\frac{d^2y}{dt^2} + \omega^2 y = 0$
(a) ω (b) $\frac{\omega}{\pi}$
(c) $\frac{\omega}{2\pi}$ (d) $2\pi\omega$

35. The equation of a harmonic oscillator is given by $\frac{d^2y}{dt^2} + ky = 0$, where k is a positive constant. What is the time period of motion?
(a) $\frac{2\pi}{k}$ (b) $2\pi k$
(c) $\frac{2\pi}{\sqrt{k}}$ (d) $2\pi\sqrt{k}$

36. The displacement equation of a harmonic oscillator is given by y = a sin ωt – b cost ωt the amplitude of the particle is
(a) a + b
(b) a – b
(c) $a^2 + b^2$
(d) $\sqrt{a^2 + b^2}$

37. A spring of force constant k is cut into three equal parts. The force constant of earth part will be
(a) k (b) 3k
(c) $\frac{k}{3}$ (d) zero

38. A particle is executing SHM with an amplitude 4 cm. At what displacement its energy is half kinetic and half potential?
(a) 2 cm (b) 1 cm
(c) $\sqrt{2}$ cm (d) $2\sqrt{2}$ cm

39. A particle executing SHM has velocity 10 cm/s and 8 cm/s at distances 4 cm and 5 cm respectively. The period of oscillation of the particle is (in seconds)
(a) π (b) 2π
(c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{2}$

40. A simple pendulum suspended from the ceiling form a lift has a period T, when the lift falls freely, the time-period of pendulum will become
(a) zero (b) $\frac{T}{9.8}$
(c) 9.8 T (d) infinity

ANSWERS

1. b	2. c	3. c	4. d
5. a	6. b	7. d	8. d
9. d	10. b	11. b	12. d
13. b	14. c	15. d	16. d
17. a	18. b	19. b	20. d
21. b	22. c	23. b	24. b
25. c	26. c	27. a	28. b
29. c	30. c	31. b	32. b
33. a	34. c	35. c	36. d
37. c	38. d	39. a	40. d

Chapter 2

WAVES / ACOUSTICS

SIMPLE HARMONC MOTION, MECMANICAL WAVES SOUND & HEARING.

SIMPLE HARMONIC MOTION (SHM)

Concepts and equation

Periodic Motion: If a moving body repeats its motion after regular intervals of time, the motion is said to be **harmonic or periodic.** The time interval after which it repeats the emotion is called **time period.** If the body moves **to and fro** on the same path, the motion is called **oscillatary.** In simple harmonic motion the particle moves in a straight line or along the angle and the acceleration of the particle is always directed towards a fixed point on the line. This fixed point is called means position or centre of oscillation. The acceleration in SHM is given by

$a = - (w^2x$ or $F = - mw^2x$ or $F = -kx$

where $k = m\omega^2$ is called force constant or spring constant.

The force which brings the particle back towards the equilibrium or mean position is called "restoring force". Such a motion is also called "isochronous".

SHM may be assumed as a **projection of uniform circular on along a diameter**

$x = r\cos\omega t$; $y = r\sin\omega t$; $a = -\omega^2 x$

or $\frac{d^2x}{dt^2} = -\omega^2 x$. This differential equation gives the solution

$x = x_o \sin\omega t$ (if the particle starts from mean position)

Fig

Fig 2.1

$x = x_o \cos\omega t$ (if the particle starts from extreme position)

$x = x_o \sin(\omega t \pm \phi)\}$ (if the particle starts in between mean and extreme position)

$x = x_0 \cos(\omega t \pm \phi))$.

The solution of differential equation in exponential form is $x = x_o^{\pm(\omega t \pm \phi)}$

Here x is instantaneous displacement, x_o is amplitude (maximum displacement), ϕ is initial phase angle or epoch or angle of repose and, ω is angular frequency.

Linear frequency $f = \frac{1}{T} = \frac{\omega}{2\pi}$ T being time period.

Velocity of the particle executing SHM

Assume $x = x_o \sin\omega t$. then $v = \frac{dx}{dt}$ $x_o\omega\cos\omega t$

$V = x_o\omega\sqrt{1-\sin^2\omega t} = \omega\sqrt{x_o^2 - x^2}$

$V_{max} = x_o\omega$; $v_{min} = 0$ at extreme position

Fig 2.2 (a) shows graph between velocity and displacement and *Fig 2.2* (b) shows the graph between velocity and time.

Fig

Fig 2.2 (a) Velocity — displacement graph

fig

Fig 2.2 (b) Velocity — time graph

Fig. 12.3 (a) and (b) shows graph between acceleration .and displacement and acceleration and time

fig

Fig 2.3 **(a)** Acceleration — displacement graph

Fig

Fig 2.3 (b) Acceleration — time graph

N.B. The graph between velocity and acceleration is an ellipse.

N.B: Velocity leads the displacement by $\frac{\pi}{2}$ but velocity lags the acceleration by $\frac{\pi}{2}$

$$a_{max}\ x_o\ \omega^2$$

$$v = x_o\ \omega \cos \omega t$$

$$\frac{dv}{dt} = - xo\ \omega^2 \sqrt{1-\cos^2 wt}$$

or $$a = - \omega^2 x,\ amax = \omega^2 x_o$$

$$a = - \omega \sqrt{(x_o\omega)^2 - (x_o\omega \cos \omega t}$$

$$a = - \omega \sqrt{v_o^2 = 1}$$

or $$\frac{a^2}{\omega^2 v_o^2} + \frac{v^2}{v_o^2} = 1$$

N.B: Velocity is manimum at mean position and acceleration is zero at mean position. Velocity is zero at extreme position and acceleration is maximum at extreme position. Kinetic energy *(KE)* of a particle executing SHM = $\frac{1}{2}\ m\omega^2\ (x_o^2 - x^2$

Potential energy *(PE)* of a particle executing SHM = $\frac{1}{2}\ m\omega^2 x^2$.

Total energy = *KE* + *PE* = $\frac{1}{2} - m\omega^2 x^2$

N.B: *KE* is maximum at mean position and zero at extreme! postion. *PE* is zero at mean position and maximum at extreme position. See Fig.2.4

Fig

Fig 2.4 KE, PE and total energy depiction.

In SHM, velocity displacement curve is an ellipse, see Fig. 2.5

Fig

Fig. 2.5 (a) Velocity displacement graph

$x = x_o \sin \omega t;$

$v = x_o\omega \cos \omega t$

or $\frac{x}{x_0} = \sin \omega t$

$\frac{x}{x_0\omega} = \cos \omega t$

Square and add (1) and (2)

$$\frac{x^2}{x_0^2} + \frac{v^2}{x_0^2\omega^2} = 1$$

acceleration – velocity relationship in SHM is an ellipse

$a = - \omega^2 x_o \sin \omega t;$

$\nu = x_o\omega \cos \omega t$

or $\frac{a^2}{\omega^4 x_0^2} = \frac{v^2}{x_0^2\omega^2}$ see Fig. 2.5 (b)

Fig

Fig 2.5 (b) Acceleration – velocity graph

If a tunnel is dug in the earth diametrically or along a chord irrespective of its position or angle then $T = 2\pi\sqrt{\frac{R}{g}}$ = 84 min 36 s for a particle released in the tunnel. See Fig. 2.6

Fig

Fig 2.6 SHM in tunnel in the earth

If a point charge q is tunneled in a uniformly charged sphere having charge Q and radius R then

$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 R^3 m}{Qq}}$$

Fig

Fig 2.7

Angular SHM A body free to rotate about a given axis can make angular oscillations when it is slightly pushed aside and released. The angular oscillations are called angular SHM.

(a) there is a mean position where the resultant torque on the body is zero ($\theta=0$).

(b) The body is displaced through an angle fro the mean position, a resultant torque α θ (angular displacement) acts.

(c) The nature of the torque (clockwise or anticlockwise) is to bring the body towards means position.

$R = - k\,\theta$ i.e

$\alpha\,\ell = - k\,\theta$ or $\alpha = - \frac{k}{\ell}\,\theta$

$\alpha = - w^2\,\theta$

$w = \sqrt{\frac{k}{\ell}}$ or $T = 2\pi\sqrt{\frac{\ell}{k}}$

solution of the equation $\alpha = - \omega^2\,\theta$ is

$\theta = \theta_o \sin \omega t$ if the particle starts from mean position

$\theta = \theta_o \sin \omega t$ if the particle starts from extreme position

$\theta = \theta_0 \sin(\omega t \pm \phi)$ if the particle starts from in between mean and extreme.

$\theta = \theta_0 \cos(wt \pm \phi)$ $\Omega = \frac{d\theta}{dt} = \theta_o\,\omega \cos \omega t$

Pendulums may be of 5 types: **simple pendulum, spring pendulum, conical pendulum, physical or compound and torsional pendulum.** Note the time period of each of them.

(fig.)

Fig 2.8 (a) Simple Pendulum

$T = 2\pi\sqrt{\frac{\ell}{g}}$ if θ is small

$T = 2\pi\sqrt{\frac{\ell}{g}}\left[1+\frac{\theta_o^{\,2}}{16}\right]$ if θ is finite and $\theta = \theta_0$

Fig 2.8 (b) Spring Pendulum

$T = 2\pi\sqrt{\frac{M}{k}}$

N.B: No effect of 'g' on spring pendulum.

Fig. 2.8 (c) Conical Pendulum

$T = 2\pi\sqrt{\frac{h}{g}}$

or $T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$

$T = 2\pi\sqrt{\frac{\ell}{mg\ell}}$

or $T = 2\pi\sqrt{\frac{k^2+1^2}{\ell g}}\,2\pi\sqrt{\frac{(k+\ell)^2-2k\ell}{\ell g}}$

Fig. 2.8 (d) Physical Pendulum

Fig

Fig. 2.8 (e) Torsional Pendulum

N.B: In physical pendulums T is maximum if $\ell = 0$ or $\ell = \infty$ and T is minimum if $k = \ell$.

Seconds pendulum: If the time period of a simple pendulum is 2s, it is called "seconds pendulum."

Longest time period (for $T = 2\pi\sqrt{\dfrac{1}{g\left(\dfrac{1}{l}+\dfrac{1}{R}\right)}}$ if $l \to \infty$ $T = 2\pi\sqrt{\dfrac{R}{g}}$ = 84 min. 36s. for an infinitely long simple pendulum) where R is radius of the earth.

If l = R, the radius of the earth then $T = 2\pi\sqrt{\dfrac{R}{2g}}$ = 60 min or 1h.

SHM under gravity: If SHM occurs due to restoring force provided by weight or acceleration due to gravity then $T = 2\pi\sqrt{\dfrac{\ell}{g}}$. Some of the examples of this type are motion of a liquid in a U-tube vertical cylinder/piston. Motion of a ball in a concave mirror/bowl and a floating cylinder as illustrated in Fig. 2.9

Fig.

fig..

(c) $T = 2\pi\sqrt{\dfrac{R}{g}}$ if ball does not roll

(d) $T = 2\pi\sqrt{\dfrac{\ell}{g}}$ but slips.

$T = 2\pi\sqrt{\dfrac{7(R-r)}{5g}}$ if the ball rolls.

Fig. 2.9

Effect of temperature on time period of simple pendulum

$\dfrac{T}{T_0} = \left[1+\dfrac{\alpha\Delta\theta}{2}\right]$ where α is linear expansion coefficient and $\Delta\theta$ negative or $\Delta T = T_o\left[\dfrac{\alpha\Delta\theta}{2}\right]$

If the upthrust of the liquid is taken into account. Then time period

$T = 2\pi\sqrt{\dfrac{l}{g\left(1-\dfrac{\sigma}{\delta}\right)}}$ and where σ is density of liquid and δ is density of the body. $a = g = g\left(1-\dfrac{\sigma}{\delta}\right)$ Damping of liquid is assumed negligible.

If the suspended wire stretches due to elasticity then time period

$$T = 2\pi\sqrt{\frac{l}{g}}\left[1+\frac{Mg}{2\pi r^2 Y}\right].$$

or $\quad \Delta T = 2\pi\sqrt{\dfrac{l}{g}}\ \dfrac{Mg}{2\pi^{r2}Y}$

or $\quad \Delta T = \dfrac{Mg}{2\pi^{r2}Y}$ where $T = 2\pi\sqrt{\dfrac{l}{g}}$ and Y is young's modulus.

If a carriage (lift) is moving up with an acceleration 'a' carrying a pendulum then

$$T = 2\pi\sqrt{\frac{l}{g+a}}$$

If the carriage (lift) moves down with an acceleration 'a' carrying the pendulum then.

$$T = 2\pi\sqrt{\frac{I}{(g-a)}}$$

If the carriage moves horizontally (e.g. a car) with an acceleration 'a' then $T = 2\pi\sqrt{\dfrac{I}{\sqrt{g^2}+a^2}}$

If the carriage is in circular motion of radius R with uniform speed v then

$$T = 2\pi\sqrt{\frac{I}{\sqrt{g^2}+\left[\frac{v^2}{r}\right]^2}}$$

If the bob of a pendulum is charged and is placed in a uniform electric field [charge q on the bob is assumed + ve in Fig. 2.10 (a) and 2.10 (b)]

fig.

fig..

Fig. 2.10

For spring system.

fig.

fig..

fig..

Fig. 2.11

Spring Pulley System:

fig.

fig..

Fig. 2.12

Fig. 2.13

Composition of two SHMs in same direction

$x_1 = x_{01} \sin \omega t$;

$x_2 = x_{02} \sin (\omega t + \theta)$

$x = x_0 \sin (\omega t + \phi) = x_1 + x_2$

$= x_{01} \sin \omega t + x_{02} \sin (\omega t + \theta)$

$x_0 = \sqrt{x_{01}{}^2 + x^2{}_{02} + 2x_{01}x_{02} \cos\theta}$

and tan ϕ

$$= \frac{x_{02} \sin\theta}{x_{01} + x_{02} \cos\theta}$$

N.B: SHMs can be added like vectors. Result is same as parallelogram Law.

Composition of two perpendicular directions give rise to lissajous figures.

$x = x_o \sin \omega t$ or $\sin \omega t \ \frac{x}{x_0}$ and $\cos \omega t$

$$= \sqrt{1 - \frac{x^2}{x_0^2}}$$

$y = y_o \sin (\omega t + \phi)$

$= y_o \sin \omega t \cos \phi + y_o \cos \omega t \sin \phi$

$$y = y_o \ \frac{x}{x_0} \ \cos \phi + y_o \sqrt{1 - \frac{x^2}{x_0^2}} \ \sin \phi$$

or $$\left(\frac{y}{y_0} - \frac{x}{x_0}\cos\phi\right)^2 = \left(1 - \frac{x^2}{x_0^2}\right) \sin^2 \phi$$

or $$\frac{y^2}{y_0^2} + \frac{x^2}{x_0^2} - \frac{2xy}{x_0 y_0} \cos \phi = \sin^2 \phi$$

if $\phi = 0$ $\left(\frac{y}{y_0} - \frac{x}{x_0}\right)^2 = 0$ or $y = \frac{y_0}{x_0}$ x, see Fig. 2.14 (a)

Fig. 2.14 (a)

If $0 < \phi = \frac{\pi}{2}$ for example $\phi = \frac{\pi}{4}$. Oblique elliqse as shown in Fig. 2.14 (b) is obtained.

Fig. 2.14 (b)

If $\phi = \frac{\pi}{2}$, elliqse is obtained and if $x_o = y_o$ the circle is obtained. See Fig. 2.15 (a) and (b)

Fig. 2.15

If $\phi = 180^o$ or π -radian then a straight line is obtained.

Fig. 2.16

Lissajous Figures: If the frequency of SHM in x-and y- direction are different then in Fig. 12.17 (a)

Fig. 2.17 (a)

$$\frac{\omega_x}{\omega_y} = \frac{n_y}{n_x}$$

= number of times it touches y-axis

number of times it touches x-axis

$$\frac{\omega_x}{\omega_y} = \frac{2.5}{1}$$

Fig. 2.17 (b)

Types of oscillations: Oscillations may be of four types

(a) Free or natural or fundamental frequency.

(b) forced

(c) resonant

(d) damped

Free or natural oscillations depend upon dimensions and nature of the material (elastic constant).

If a periodic force of frequency other than the natural frequency of the material is applied then **forced oscillations result.**

For example if $y = y_0 \sin wt$ was the equation of SHM of a particle and a periodic force $p \sin \omega_1 t$ is applied ($\omega \neq \omega_1$)then $y = y_0 \sin \omega t + p \sin \omega_1 t$. The resultant frequency is different from natural frequency of oscillation

Resonant oscillation are a special kind of forced oscillation in which frequency of the source = frequency of the applied force, i.e., $y = y_0 \sin \omega t + p \sin \omega t = (y_0 + p) \sin \omega t$. That is amplitude increases or intensity increases or intensity increases with resonance. In damped oscillations amplitude of vibrations falls with time as shown in Fig. 2.18.

Fig.

Fig. 2.18

Amplitude at any instant is given by = $y = y_0 e^{-bt}$ where is amplitude of first vibration and y is amplitude at time t and is damping coefficient.

Damped harmonic motion

$$\frac{md^2x}{dt^2} + r\frac{dx}{dt} + kx = 0$$

or $$\frac{d^2x}{dt^2} + \frac{r}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

or $$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega^2 x == 0$$

where $b = \frac{r}{2m}$ is called damping coefficient.

$$x = \frac{x_0}{2} e^{-bt}\left[\left(1+\frac{b}{\sqrt{b^2-\omega^2}}\right)e^{t\sqrt{b^2-\omega^2}}\right]$$

$$+\left[\left(1-\frac{b}{\sqrt{b^2-\omega^2}}\right)e^{-1\sqrt{b^2-\omega^2}}\right]$$

gives amplitude at any instant.

If $\frac{r}{2m} > \sqrt{\frac{k}{m}}$ or $b > \omega$ motion is over damped and non-oscillatory.

If $\frac{r}{2m} = \sqrt{\frac{k}{m}}$ or $b = \omega$ motion is critically damped and $x = x_0 e^{-bt}$

If $\frac{r}{2m} < \sqrt{\frac{k}{m}}$ or $b < \omega$ damped oscillatory motion with time period

$$T = \frac{2\pi}{\sqrt{\omega^2 - b^2}} = \frac{2\pi}{\sqrt{\frac{k}{m} - \frac{r^2}{4m^2}}}$$

If r = 0 motion is undamped and $T = 2\pi\sqrt{\frac{k}{m}}$

Wave Motion and Wave in a String

Three types of waves may be defined — mechanical, electromagnetic and matter waves as illustrated in Fig. 2.19. Here we are concerned with mechanical waves only.

Fig. 2.19 Waves description

A Wave is a disturbance which propagates energy from one place to the other without transporting matter. It is spread over region without clear-cut boundaries. It is not localized.

Diffraction It is a convincing proof of wave nature. It differentiates between particle nature and wave nature.

Mechanical waves require an elastic medium to progagate. Therefore, mechanical waves are also called **elastic waves.** Waves like electromagnetic and matter waves do not require any medium to propagate.

Shock waves are a kind of pulse propagation and are mathematically expressed as

$$y = \frac{a}{b + (x \pm vt)^2}$$

Shock waves are produced during earthquakes, volcanic eruptions, bomb blasts and during a sonic boom.

$y = y_0\ (\omega t - kx)$ is the wave propagating along positive *x* direction.

Plane progressive wave is given by

$$y = y_0 \sin (\omega t - kx)$$

where *k* is called "propagation constant" or "wave number," ω is called "angular frequency," y_0 amplitude and v instantaneous displacement. Such a wave is called a "displacement wave."

$$K = \frac{2\pi}{\lambda} \text{ where } \lambda \text{ is wavelength,}$$

$(\omega t - kx)$ is the phase at any instant. When path difference $\Delta x = \lambda$, then phase shift $\Delta\phi = 2\pi$. In general $k\ \Delta x = \Delta\phi$.

A wave can have two types of velocities.

Wave velocity or phase velocity and group velocity or particle velocity

Wave velocity $v = \frac{dx}{dt} = \frac{w}{k} = f\lambda$

In a dispersive medium, wave travels with a group velocity

$$v_{group} = v - \lambda \frac{dv}{d\lambda}$$

MCQs

MECHANICAL WAVES, SOUND AND HEARING

1. A disturbance that travels through space and time usually accompanied by energy transfer is called
   (a) wave
   (b) sound
   (c) frequency
   (d) length

2. A wave that consists of oscillation occurring perpendicular to the direction of energy transfer is called
   (a) transverse wave
   (b) longitudinal wave
   (c) stationary wave
   (d) shock wave

3. Wave that have the same direction of vibration as their direction of travel are termed
   (a) longitudinal waves
   (b) traverse waves
   (c) standing waves
   (d) hair waves

4. A water wave is an example of
   (a) transverse wave
   (b) longitudinal wave
   (c) stationary wave
   (d) shock wave

5. A sound wave is an instance of
   (a) transverse wave
   (b) longitudinal wave
   (c) hair wave
   (d) stationary wave

6. Which of the following is a transverse wave?
   (a) sound wave
   (b) shock wave
   (c) hair wave
   (d) radio wave

7. A wave that remains in a constant position is called
   (a) standing or stationary wave
   (b) transverse wave
   (c) shock wave
   (d) longitudinal wave

8. On average, there is no energy transfer in
   (a) sound waves
   (b) water waves
   (c) standing waves
   (d) mechanical waves

9. Which of the following refers to the distance from crest to crest of a wave?
   (a) frequency
   (b) wavelength
   (c) amplitude
   (d) period

10. The maximum displacement from the undisturbed position of the medium to the crest top is called
   (a) wavelength
   (b) amplitude
   (c) period
   (d) frequency

11. When waves go from one place to another, they transport
   (a) wavelength
   (b) period

(c) energy
(d) amplitude

12. The energy transported by a wave is proportional to the square of the ———— of the wave.
(a) wavelength
(b) period
(c) amplitude
(d) frequency

13. A mechanical wave is a disturbance that travels through some material or substance called the ———— for the wave
(a) period
(b) medium
(c) frequency
(d) amplitude

14. The average amount of energy transported by a wave, per unit area per unit time is termed as
(a) wave speed
(b) wave intensity
(c) wavelength
(d) wave amplitude

15. The phase velocity is the velocity of a point that moves with a wave at constant phase. It is also called
(a) phase speed
(b) wave speed
(c) wave celerity
(d) all of the above

16. The addition of two or more waves is termed ————
(a) interference
(b) amplitude
(c) period
(d) frequency

17. The human ear is sensitive to waves in the frequency range also called the audible range from about 20 Hz to
(a) 200 Hz
(b) 2000 Hz
(c) 20,000 Hz
(d) 2,00,000 Hz

18. The perceived fundamental frequency of a sound is called
(a) loudness
(b) timber
(c) pitch
(d) none of these

19. The loudness of a sound depends on its
(a) frequency
(b) amplitude
(c) both a and b
(d) neither a nor b

20. Which technique uses underwater sound propagation to detect and locate submerged objects?
(a) SONAR
(b) RADAR
(c) LIDAR
(d) none of the above

21. Sonography is an ultra-sound based imaging technique used for diagnosis. It uses sound waves of
(a) less than 20 kHz
(b) 20 kHz
(c) greater than 20 kHz
(d) none of these

22. The speed of sound is air at 20°C is
(a) 344 m/s
(b) 1402 m/s
(c) 1482 m/s
(d) 1543 m/s

23. During WWI, which device was used by submarines to detect targets while submerged?
(a) SONAR
(b) Microphone
(c) Hydrophone
(d) RADAR

24. Hearing damage is possible at sound pressure of
    (a) 0 dB (b) 50 dB
    (c) 130 dB (d) 195 dB

25. Which of the following devices is used for playing sound recordings? (The other name of gramophone)
    (a) Phonograph
    (b) Headphone
    (c) Microphone
    (d) Amplifier

26. For polarization, the direction of oscillation has to be perpendicular to the direction of travel. Sound waves are longitudinal waves so they cannot be
    (a) reflected
    (b) deflected
    (c) diffracted
    (d) polarized

27. Transverse waves only propagate in
    (a) liquids (b) solids
    (c) gases (d) all of the above

28. Wave motion in air consist of
    (a) longitudinal waves
    (b) transverse waves
    (c) seismic waves
    (d) polarized waves

29. In an oscillating system, damping is reduction in
    (a) frequency
    (b) wavelength
    (c) amplitude
    (d) period

30. Which physical property is most responsible for resonance?
    (a) frequency
    (b) intensity
    (c) pitch
    (d) loudness

31. A sonometer or audiometer is a device based on the principle of
    (a) resonance
    (b) beats
    (c) overtones
    (d) harmony

32. A sonometer is also called "monochord" because it often has
    (a) one string
    (b) two strings
    (c) three strings
    (d) four strings

33. An ———— is any frequency higher than the fundamental frequency of a sound
    (a) overtone (b) beat
    (c) acoustics (d) shockwave

34. Which field of science deals with the study of all mechanical waves including vibrations sound?
    (a) electronics
    (b) acoustics
    (c) robotics
    (d) statistics

35. A ———— is an interference between two sounds of slightly different frequencies
    (a) shockwave
    (b) beat
    (c) sonic boom
    (d) none of them

36. The amplitude of a vibrating body at resonance placed is vacuum is
    (a) zero (b) maximum
    (c) minimum (d) infinite

37. Beats occur due to
    (a) reflection
    (b) refraction
    (c) interference
    (d) none of the above

38. The fixed ends of a vibrating string are
    (a) overtones
    (b) nodes
    (c) antinodes
    (d) none of the above

39. How many antinodes must be there between two nodes?
    (a) 1 (b) 2
    (c) 3 (d) 4

40. The note of the lowest frequency is called the
    (a) beat
    (b) overtone
    (c) fundamental note
    (d) none of the above

41. What is the velocity of sound in vacuum?
    (a) 768 mph
    (b) zero
    (c) 3136 mph
    (d) 3315 mph

42. A bomb explodes on the Mars. How long it will take for the sound to reach the earth?
    (a) 8 min
    (b) 10 sec
    (c) 0 sec
    (d) 1 day

43. Two waves of same frequency having amplitudes a and 2a travelling in the same direction superimpose out of phase. What will be the resultant amplitude?
    (a) a (b) 2a
    (c) 3a (d) $\sqrt{a^2 + 2a^2}$

44. Which type of oscillations produce resonance?
    (a) free (b) damped
    (c) forced (d) none of these

45. The ratio of intensities of two waves is 1:16. What will be the ratio of their amplitudes?
    (a) 1 : 1 (b) 1 : 2
    (c) 1 : 4 (d) 1 : 16

46. What is the distance between a node and an antinode?
    (a) λ (b) $\lambda/2$
    (c) $\lambda/4$ (d) 2λ

47. The phase difference between the particles vibrating between two consecutive nodes is
    (a) zero (b) $\frac{\pi}{2}$
    (c) π (d) 2π

48. The ratio of frequencies in a stretched string is
    (a) 1 : 2 : 3 (b) 1 : 3 : 5
    (c) 3 : 2 : 1 (d) 2 : 4 : 6

49. What is the change in path, when sound wave is reflected from a rigid support?
    (a) 2π (b) π
    (c) $\pi/2$ (d) zero

50. What is the term used for the ratio of the speed of a body and speed of sound?
    (a) Avogadro's number
    (b) Mach number
    (c) Feigenbum number
    (d) Telephone number

51. In which of the following the speed of sound will be maximum under similar conditions?
    (a) N_2 (b) O_2
    (c) H_2 (d) CO_2

52. On which characteristics, the loudness of sound depends on?
    (a) pitch
    (b) amplitude
    (c) speed

(d) wavelength

53. The ratio of intensities of two sound waves is 4 : 9. What will be the ratio of their amplitudes?
    (a) 9 : 4 (b) 3 : 2
    (c) 2 : 3 (d) 4 : 9

54. What is the best sound source to produces a pure note?
    (a) flute
    (b) tuning fork
    (c) harmonium
    (d) drum

55. In order to hear an echo, what is the minimum distance between the sound and reflecting surface?
    (a) 0.65 m (b) 1.65 m
    (c) 16.5 m (d) 165 m

56. What will be the sound speed if the frequency is doubled?
    (a) zero (b) half
    (c) double (d) unchanged

57. What is the shape of a pure tone?
    (a) sinewave
    (b) square wave
    (c) sawtooth
    (d) triangular wave

58. Which acoustical apparatus is used for measurement of the speed of sound in a gas or a solid rod?
    (a) Melde's experiment
    (b) Kundt's tube
    (c) Michelson-Morley experiment
    (d) Robert Milkan's oild-drop experiment

59. For which waves phenomenon of beats takes place?
    (a) longitudinal waves
    (b) transverse waves
    (c) both longitudinal and transverse waves
    (d) none of the above

60. A pendulum vibrates with a time period of 1s. Which range of sound is produced by it?
    (a) audible
    (b) infrasonic
    (c) ultrasonic
    (d) supersonic

61. Which characteristic successively increases in the musical scale?
    (a) quality
    (b) pitch
    (c) loudness
    (d) amplitude

62. If A is the amplitude of sound wave after covering a distance r, then
    (a) $A \alpha \frac{1}{r^2}$ (b) $A \alpha \frac{1}{r}$
    (c) $A \alpha r$ (d) $A \alpha r^2$

63. What will be the frequency if an empty vessel is filled with water?
    (a) increases
    (b) decreases
    (c) remains unchanged
    (d) none of the above

64. What is title for combination of notes that produce jarring effect on the ear?
    (a) noise (b) melody
    (c) harmony (d) discord

65. Mostly human ear cannot hear sound of intensity less than
    (a) 10^{-3} W/m^2 (b) 10^{-6} W/m^2
    (c) 10^{-12} W/m^2 (d) 10^{-15} W/m^2

66. What is term for the persistence of sound in a hall?
    (a) resonance
    (b) acoustics
    (c) symphony
    (d) reverberation

67. Which effect produces ultrasonics in quartz?
    (a) Pyroelectric effect
    (b) Piezoelectric effect
    (c) Hall effect
    (d) Magnetostriction effect

68. Which phrase is used for reproduction of original sound?
    (a) loyalty
    (b) obedience
    (c) fidelity
    (d) conformity

69. Which analysis is employed to convert a complex sound into notes?
    (a) Fourier theorem
    (b) Milleman theorem
    (c) Lissajoes theorem
    (d) Demrogan's laws

70. Which law states that a musical sound is perceived by the ear as a set of a number of constituent pure harmonic tones?
    (a) Kirchoff's law
    (b) Ohm's acoustic law
    (c) Faraday's law
    (d) Hopeinson's law

71. The velocity of sound in water is ———— than that of air
    (a) smaller
    (b) greater
    (c) unchanged
    (d) none of the above

72. The beat frequency is the ———— of the two frequencies.
    (a) sum (b) product
    (c) difference (d) ratio

73. Which effect explains the frequency shift that occurs when there is motion of sound, a listener or both relative to the medium?
    (a) Early effect
    (b) Doppler effect
    (c) Einstein effect
    (d) Hall effect

74. Besides sound, Doppler effect occurs for
    (a) light
    (b) radio waves
    (c) both a and b
    (d) none of the above

75. Which of the following is a mechanical wave?
    (a) x-rays
    (b) radio waves
    (c) light
    (d) sound

76. Which property of sound is not affected by change in air temperature?
    (a) amplitude
    (b) wavelength
    (c) intensity
    (d) frequency

77. As a man moves away form a steady source of sound at constant speed, the sound he hears will
    (a) increase in frequency and intensity
    (b) stay constant in pitch but decrease in loudness
    (c) increase in frequency and intensity
    (d) constant in both pitch and loudness

78. When the source and observer are moving away from each other the apparent pitch will
    (a) increase (b) decrease
    (c) be zero (d) be infinite

79. When wind blows in the same direction in which the sound travels, the sound velocity
    (a) decreases
    (b) increases
    (c) remains constant

(d) none of the above

80. When the prong of a tuning fork is loaded with waxe, its frequency
    (a) increases
    (b) decreases
    (c) remains constant
    (d) none of the above

81. Who developed gramophone is 1877?
    (a) Thomas Edison
    (b) Graham Bell
    (c) Isaac Newton
    (d) Michael Faraday

82. What is the linear succession of musical tones that is perceived as a ring entity?
    (a) harmony (b) noise
    (c) melody (d) music

83. For how long the sensation of sound persists in our brain?
    (a) 0.4 sec (b) 0.3 sec
    (c) 0.2 sec (d) 0.1 sec

84. Which of the following is a conical-bore transposing musical instrument that is a member of the wood-wind family?
    (a) telephone
    (b) gramophone
    (c) saxophone
    (d) cell phone

85. What is a sound recording device used for recording speech for later playback or to be typed into print?
    (a) saxophone
    (b) gramophone
    (c) telephone
    (d) dictaphone

86. Due to which reason, echoes arise?
    (a) refraction (b) diffraction
    (c) reflection (d) dispersion

87. What is a sequence of musical notes in ascending and descending order that provides material for representing musical work?
    (a) Beaufort scale
    (b) Richter scale
    (c) Newton scale
    (d) Musical scale

88. Which two-pronged metal device when struck, produces a sound of constant pitch?
    (a) force
    (b) tuning fork
    (c) samophone
    (d) dictaphone

89. On which parameter, the path difference between two interfering waves depend upon?
    (a) amplitude (b) pitch
    (c) intensity (d) phase angle

90. To what forms of waves the phenomenon of interference applies?
    (a) sinusoidal
    (b) square
    (c) triangle
    (d) all of the above

91. Which of the following represents an elastic wave?
    (a) light waves
    (b) radio waves
    (c) x-rays
    (d) sound waves

92. When two identical (progressive) waves are super-imposed, the velocity of resultant wave
    (a) increases
    (b) decreases
    (c) becomes zero
    (d) remains unchanged

93. Which of the following properties of sound is affected by change in air temperature?
    (a) frequency
    (b) amplitude

(c) intensity
(d) wavelength

94. What is the particle velocity at the nodal points in a stationary wave?
(a) zero
(b) maximum
(c) minimum
(d) none of the above

95. What is the distance between two consecutive modes or antinodes in a stationary wavelength
(a) λ (b) $\frac{\lambda}{2}$
(c) $\frac{\lambda}{4}$ (d) $\frac{\lambda}{8}$

96. For every 1°C rise in temperature, the sound velocity
(a) increases by 61 cm/s
(b) decreases by 61 cm/s
(c) remains constant
(d) none of the above

97. The sound, velocity in moist air as compared to dry air will be
(a) more
(b) less
(c) same
(d) none of the above

98. Two sources of sound are said to be in resonance when,
(a) they look like similar
(b) they produce sound of same frequency
(c) they are enacted by the same agent
(d) none of the above

99. Beats are the result of
(a) diffraction of sound waves
(b) constructive and destructive interference
(c) both of the above
(d) none of the above

100. Which of the following property differentiates between transverse and longitudinal waves?
(a) interference
(b) diffraction
(c) polarisation
(d) any one of the above

101. A pulse on the string is inverted when it is reflected from
(a) fixed end
(b) free end
(c) free or fixed end
(d) none of the above

102. A body travels with a speed greater than the speed of sound. What would be the wavefront shape?
(a) elliptical
(b) spherical
(c) parabolical
(d) conical

103. Sweetness of sound depends on
(a) amplitude
(b) frequency
(c) velocity
(d) periodicity and regularity

104. The same notes being played on sitar and veena differ in
(a) pitch
(b) quality
(c) both quality and pitch
(d) neither quality nor pitch

105. When temperature increases, frequency of organ pipe
(a) decreases
(b) remains same
(c) increases
(d) becomes zero

106. An observer and a sound source are moving away from each other. The apparent pitch will
(a) remain the same
(b) increase

(c) decrease
(d) none of the above

107. A star emitting yellow light starts accelerating towards the earth. Its colour as seen from the earth will turn gradually
(a) red (b) violet
(c) green (d) blue

108. Doppler's principle applies to
(a) sound waves only
(b) light waves only
(c) neither sound nor light waves
(d) both sound and light waves

109. Two tuning forks have same natural frequency. One of them is now loaded with wax. When both the forks are sounded together, they will
(a) produce interference
(b) produce vibrations
(c) remain in resonance
(d) produce beats

110. In which of the following the speed of sound is greatest?
(a) air (b) ammonia
(c) water (d) steel

111. The wavelength of ultrasonic waves in air is of the order of
(a) 1 cm (b) 10^{-2} cm
(c) 10^{-4} cm (d) 10^{-8} cm

112. On the average, the maximum number of syllabus uttered by a human being per second does not exceed
(a) 2 (b) 5
(c) 8 (d) 11

113. Three tuning forks of frequencies 400, 401 and 402 Hz are sounded together. The frequency of beats per second is
(a) 0 (b) 1
(c) 2 (d) 3

114. 56 tuning forks are so arranged in series, that each fork gives 4 beats/second with the previous one. The frequency of the last fork is thrice that of the first. What is the frequency (in Hz) of the last fork?
(a) 52 (b) 56
(c) 60 (d) 110

115. The fundamental frequency of a sound source is 256 Hz. What is the frequency of its first harmonic?
(a) 128 Hz (b) 64 Hz
(c) 512 Hz (d) 256 Hz

116. Two waves represented by $y_1 = a_1$ sin wt and $y_2 = a_2$ cost wt are superimposed at any point at a particular instant. What is the amplitude of the resultant wave?
(a) $a_1 + a_2$ (b) $a_1 - a_2$
(c) $\sqrt{a_1^2 - a_2^2}$ (d) $\sqrt{a_1^2 + a_2^2}$

ANSWERS

1. a	2. a	3. a	4. a
5. b	6. d	7. a	8. c
9. b	10. b	11. c	12. c
13. b	14. b	15. d	16. a
17. c	18. c	19. c	20. a
21. c	22. a	23. c	24. c
25. a	26. a	27. b	28. a
29. c	30. a	31. a	32. a
33. a	34. b	35. b	36. d
37. c	38. b	39. a	40. c
41. b	42. c	43. a	44. c
45. c	46. c	47. a	48. a
49. c	50. b	51. c	52. b
53. c	54. b	55. c	56. d
57. a	58. b	59. c	60. b
61. b	62. b	63. a	64. d
65. c	66. d	67. b	68. c
69. a	70. b	71. b	72. c
73. b	74. c	75. d	76. d
77. c	78. b	79. b	80. b
81. a	82. c	83. d	84. c
85. d	86. c	87. d	88. b
89. d	90. d	91. d	92. d
93. d	94. a	95. b	96. a

97. a	98. b	99. b	100. c	109. d	110. d	111. a	112. c
101. a	102. d	103. d	104. b	113. b	114. d	115. d	116. d
105. c	106. c	107. d	108. d				

Chapter 3

OPTICS

CONCEPTS AND EQUATIONS

Reflection

Rebounding of light from a polished surface like a mirror is called "reflection."

Laws of Reflection

(a) Angle of incidence = angle of reflection.

(b) Incident ray, normal and reflected ray are coplanar.

If mirror is rotated by θ, reflected ray moves by 2θ.

Fig.

Fig. 3.1 Reflection from a polished surface

Diffusion

Reflection from a rough surface (such as a wall) is called "diffusion." A parallel beam will not emerge out parallel after suffering reflection because it meets different angles at the reflecting surface (see Fig. 3.2)

Characteristics of image formed with a plane mirror

(a) It is erect

(b) It is virtual

(c) Size of image = Size of object.

(d) Image distance = Object distance (measured from mirror).

(e) Lateral inversion (left appears right and right appears left).

Fig.

Fig. 3.2 Illustration of diffusion

Number of Images

If two mirrors are inclined at an angle θ the number of images formed for an object placed in front of them is given by

(a) Number of images $n = \frac{360}{\theta}$ if $\frac{360}{\theta}$ is odd and object does not lie on angle bisector or is placed symmetrically. $n = \frac{360}{\theta} - 1$ if $\frac{360}{\theta}$ is odd and object is placed on angle bisector.

(b) Number of images $n = \frac{360}{\theta} - 1$ if $\frac{360}{\theta}$ is even (object placed non-symmetric). $n = \frac{360}{\theta}$ if $\frac{360}{\theta}$ is even (object placed symmetrically).

If two mirrors are parallel ($\theta - 0$) $n = \infty$.

A, H, I, M, O, ….. U, V, X, Y etc. 11 letters show lateral symmetry.

If mirror is thick, second image (formed due to first reflection from polished surface) is the brightest.

When a ray is reflected from a plane mirror, angle of deviation is $\delta = \pi - 2\theta$ as shown in Fig. 3.3.

Fig.

Fig. 3.3 Finding angle of deviation

Minimum height of a mirror so that a person can see his full image in the mirror is half the height of the mirror when standing at a distance = half the height away from the mirror.

Spherical mirrors are of two types

Convex and concave as shown in Fig. 3.4.

Fig.

Fig. 3.4 (a) Concave and (b) Convex mirror

Fig.

Fig. 3.5 Illustration of sign convention

Sign Convention

Consider pole P as origin. All distances to its left are negative and all distances to its right are positive.

Mirror Formulae

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \text{ and } f = \frac{R}{2} \text{ where}$$

v = image distance from pole to mirror

u = object distance from pole to mirror

f = focal length

R = radius of curvature

Table 3.1

Real Image	Virtual Image
1. Rays actually converge to form image.	Rays appear to diverge from image
2. Image can be obtained on screen.	Image cannot be taken as screen.
3. Image is inverted.	Image is erect.
4. Magnification is negative	Magnification is positive.

Magnification M_{lat} (lateral) or linear magnification.

$$M_{lat} = \frac{I}{o} = \frac{-v}{u} = \frac{v - f}{f} = \frac{f}{u - f}$$

See Fig. 3.6(a)

Fig.

Fig. 3.6(a) Lateral magnification

Magnification (axial) $M_{axial} = \frac{-v^2}{u^2}$

(used for small objects only).

See Fig. 3.6(b).

Fig.

Fig. 3.6(b) Axial magnification

Lens

The part of an isotropic transparent medium bounded by at least one curved surface. Lenses are of two types (a) convex (b) concave.

Fig.

Fig..

Fig. 3.7

Table 3.2 Image formation information for convex lens and concave mirrors

Position of object	Position of image and its nature
At ∞	At focus (real, inverted, diminished)
Away from 2f	Between f and 2f (real, inverted and diminished).
	At 2f (real, inverted and equal in size)
Between f and 2f	Away from 2f, (real, inverted and magnified). At ∞ (real, inverted and magnified)
Between pole and f	Behind the mirror (virtual erect and magnified). In front of lens, i.e., on the same side of object.

Remember spherical mirrors have principal focus while lenses have two principal focii one on each side as shown in Fig. 3.8.

Fig.

Fig. 3.8 Illustration of principal foci in a lens

Lens formulae for thin lenses

$\frac{1}{f} = (\mu_2 - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ (lens maker's formula) when surrounding medium is air or vacuum.

$$\frac{1}{f}\left(\frac{\mu_2}{\mu_m} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

if surrounding medium has refractive index μ_m.

Lens formula

$\frac{1}{v} - \frac{1}{\mu} = \frac{1}{f}$ (in air or vacuum)

Fig.

Fig. 3.9 Displacement method to find focal length of a convex lens

Displacement method

If object and image (or screen) are fixed at a distance D(> 4f) [Fig. 3.9]. Lens is set at L_1 to form a magnified sharp image at I_0. Then lens is displaced by d again to form a sharp image at I (diminished). Then $f = \frac{D^2 - d^2}{4d}$ and $O = \sqrt{I_1 I_2}$ where I_1 and I_2 are sizes of image in magnified and diminished position of lens L_1 and L_2 respectively O is size of object.

Lateral magnification

$M_{lateral} = \frac{v}{u} = \frac{1}{O}$ for a convex lens.

$M_{lateral} = \frac{-v}{u} = \frac{1}{O}$ for a concave lens

$$M_{lateral} = \frac{f}{u + f} = \frac{f - v}{f}$$

Axial magnification

$M_{axial} = \frac{-v^2}{u^2}$ (for small objects)

If object and image are formed in different media then use

$\frac{\mu_2}{f} = \frac{\mu_2 - \mu_1}{R_1} - \frac{\mu_2 - \mu_3}{R_2}$ to find focal length

$\frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} - \frac{\mu_2 - \mu_3}{R_2}$ to find v or u

Fig.

Fig. 3.10 Image formation when lens lies in two different media

Fig.

Fig. 3.11 Combination of two thin lenses

If two thin lenses are in contact as shown in Fig. 3.11 then $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

Newton's formula

$x_1 x_2 = f_2$ [Fig. 3.12]

Fig.

Fig. 3.12 Focal length using Newton's formula

If focal length on two sides is not equal then $f_1 f_2 = x_1 x_2$ (in case O and I are in different mediums)

Fig.

Fig. 3.13 Combination of lenses when at a distance d apart

If two lenses are distance d apart as shown in Fig. 3.12 then their combined focal length $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

Focal length of a thick lens of thickness f

$$\frac{1}{f} = (\mu_2 - 1)\left[\frac{1}{R_1} - \frac{1}{R_2} - \frac{t(\mu_2 - 1)}{\mu_2 R_1 R_2}\right]$$

Cardinal points

There are three sets of cardinal points.

(a) Set of focal points.

(b) Principal points $x_1x_2 = f_2$ (if lens is in air)

(c) Nodal points

Flare spots

If strong light is used, more than one refraction occurs in a lens and hence more than one image is formed called spots. For nth flare spot.

$$\frac{1}{f_n} = \frac{(n+1)\mu - 1}{f(\mu - 1)}$$

Power of the lens

$P = \frac{1}{f(m)} = \frac{100}{f(cm)}$. This unit is dioptre (D).

Defects in lenses

(a) **Spherical aberration** (or monochromatic aberration): occurs as paraxial and marginal rays fail to meet at a point as illustrated in Fig. 3.14. Spherical aberration can be removed using optical stops or aplanatic lens. Astigmatism is cured by cylindrical lens.

Fig.

Fig. 3.14 Spherical aberration illustration

(b) **Chromatic aberration** A white object when seen through a lens appears coloured. Such a defect is called "chromatic aberration." Its removal is called "achromatism." For achromatic aberration a combination of a convex and a concave lens is needed such that $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$ where ω_1 and ω_2 are dispersive powers of two lenses of focal length f_1 and f_2 respectively. Their focal length is

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$. See Fig. 3.15

Fig.

Fig. 3.15 (a) Achromat combination

Achromatic aberration can also be removed using two lenses of same kind separated by a small distance if

Fig.

Fig. 3.15 (b) Achromatism using two convex lens

$d = \frac{\omega_1 f_2 + \omega_2 f_1}{\omega_1 + \omega_2}$ as illustrated in Fig. 3.15(b).

Note: If $\omega_1 = \omega_2$ then $d = \frac{f_1 + f_2}{2}$.

If $d = f_1 - f_2$ spherical aberration is also removed.

Thus if $f_1 = 3f_2$ and $d = 2f_2$ then both the defects can be removed simultaneously. This approach is employed in Huygen's eye piece.

Refraction

When an oblique ray of light enters from one medium to another (optically different or dispersive medium) then it changes its path. Such a phenomenon is called "refraction." (See Fig. 3.16).

Fig.

Fig. 3.16 Refraction in a dispersive medium

N.B2 It does not mean that if the ray is incident normal, it is not refracted.

Laws of Refraction

There are two laws of refraction.

(a) $1\mu_2$ or $\mu = \frac{\sin i}{\sin r}$

(b) Incident ray, normal and refracted rays are coplanar.

$\mu = \frac{\sin i}{\sin r} = \frac{c}{v}$ or $\frac{v_1}{v_2} = \frac{1}{\sin C}$ where C is critical angle.

$\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$ (Apply this formula when incidence is normal)

$\mu = \frac{\lambda_1}{\lambda_2} = \tan \theta_P$ where θ_P is polarising angle and is equal to angle of incidence if angle between reflected and refracted rays is 90°.

$$\mu = \frac{\sin \frac{A + D_m}{2}}{\sin \frac{A}{2}}$$ in a prism.

$\delta = (\mu - 1)\,\alpha$ where α is angle of prism and δ is angle of minimum deviation in a prism of small angle α (angle of prism).

$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^2}$ is called Cauchy's principle.

${}^1\mu_2 = \frac{1}{{}^2\lambda_l}$ (principle of reciprocity)

${}^1\mu_3 = {}^1\mu_2 \times {}^2\mu_3$

Fermat's Principle

When a ray of light passes from one point to another by any number of reflections or refractions, the path taken by the light is the one for which corresponding time taken is the least (or has shortest optical path).

Optical path length is μl if l is the distance travelled in a medium of refractive index μ.

$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{u_2 - \mu_1}{R}$ (See Fig. 3.19)

Fig.

Fig. 3.17 Refraction through a curved surface

Note that $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ can be applied for all curved surfaces with appropriate sign convention and remembering that μ_1 is the refractive index of the medium in which object lies.

Dispersion

Splitting of a complex light into its constituent colours is called "dispersion." For example, white light splits into seven colours when passed through a prism.

In a prism i + e = A + D (See fig.3.18).

Fig.

Fig. 3.18 Refraction through a prism

Fig. 3.19 shows graph between angle of deviation D and angle of incidence i. D_m is angle of minimum deviation.

Fig.

Fig. 3.19

At minimum deviation i = e and $r_1 = r_2$. The ray through the prism is a parallel to the base of the prism.

Under minimum deviation condition

$$\mu = \frac{\sin \frac{A + D_m}{2}}{\sin \frac{A}{2}}.$$

Dispersive Power

$$\omega^1 = \frac{\delta_v - \delta_r}{\delta} = \frac{\mu_v - \mu_r}{\mu - 1}$$

where δ_v and δ_r are minimum deviations for violet and red colours, δ is mean deviation (for yellow colour). μ_v and μ_r are the refractive index for violet and red colours and μ is the refractive index for yellow or mean colour.

Note: Use $\delta =$ if δ is not given. Similarly use

$$\mu = \frac{\mu_v + \mu_r}{2} \text{ if } \mu \text{ is not given.}$$

$\omega\delta = \delta_v - \delta_r$ is called angular dispersion.

Rainbow

Two types of rainbows are known: primary rainbow and secondary rainbow.

Primary rainbow is formed when one total internal reflection (TIR) and two refractions occur from the suspended raindrops as illustrated in Fig. 3.20(a). Violet colour on inner edge and red colour on outer edge are seen, as shown in Fig. 3.20(c). Angles subtended with the direction of sun are 42° (violet) above the horizon.

Fig.

Fig. 3.20(a) Primary rainbow formation

Fig.

Fig. 3.20(b) Secondary rainbow formation

Secondary rainbow is formed due to two TIRs and two refractions from the raindrops suspended in air as shown in Fig. 3.20(b). Inner edge has red colour and outer edge violet. i.e., there is colour reversal from primary rainbow. It occurs due to an additional reflection which causes 180° phase shift. Angles are 51° for red and 54° for violet.

Fig.

Fig. 3.20(c) Rainbow

Deviation without dispersion

See Fig. 3.21(a)

Condition $(\delta_{v_1} - \delta_{r_1}) = (\delta_{v_2} - \delta_{r_2})$

or $(\mu_{v_1} - \mu_{r_1})\,\alpha_1 = (\mu_{v_2} - \mu_{r_2})\,\alpha_2$

or $\delta_1\omega_1 = \omega_2\delta_2$

Dispersion without deviation

See Fig. 3.21(b). The mean colour should be parallel to incident ray.

$(\mu_1 - 1)\,\alpha_1 = (\mu_2 - 1)\,\alpha_2$

Fig.

Fig. 3.21(a) Deviation without dispersion

Fig.

Fig. 3.21(b) Dispersion without deviation

The prisms which produce dispersion without deviation are called "direct vision prism" and are employed in direct vision spectroscope. If more than two prisms are used the resolving power of the spectroscope is increased.

Defects in Human Eye

1. Myopia or short-sightedness
2. Hypermetropia or long sightedness
3. Presbyopia
4. Astigmatism
5. Colour blindness

1. **Myopic** eye is treated by concave lens. (Image is formed in front of the retina).
2. **Hypermetropic** eye is treated by convex lens. (Image is formed beyond the retina).
3. **Presbyopia** eye with this defect can neither see near objects nor far objects clearly. It is treated by bifocal lens (upper half concave and lower half convex).
4. **Astigmatism** is treated by specially prepared cylindrical lens.
5. **Colour blindness** eye cannot differentiate between colours. Remedy is not available.

An alternative approach for correcting many defects of vision is to reshape the cornea. It is done using a procedure called Laser assisted in situ Keratomileusis or LASIK. An incision is made into the cornea and a flap of outer corneal tissue is folded back. A pulsed uv laser with a beam only 50 μm wide $\left(< \frac{1}{200}\text{th width of the hair} \right)$ is then used to vaporise away microscopic area underlying the tissue. The flap is then folded back to the position where it conforms to the new shape carved by the laser.

Visual acuity or resolving power of eye is $\frac{1}{60^\circ}$ or 1 min.

Near point is 15 cm and least distance of distinct vision (normal near point) = 25 cm.

Eye Pieces or Occular

Commonly used eyepieces are Huygen's and Ramsden. In Huygen eyepiece both the defects, spherical aberration and chromatic aberration, are removed. If f_1 and $3f_2$ and $d = 2f_2$ then $d = \frac{f_1 + f_2}{2}$ removes chromatic aberration and $d = f_1 - f_2$ removes spherical aberration. The drawback in Huygen's eyepiece is that crosswire cannot be fitted. Therefore it can be used for qualitative work. Wherever quantitative (measurements) work is involved Ramsden's species is used. Remsden eyepiece comprises of two lenses of equal focal length. $d = \frac{2}{3}$ f. It is achromated for two selected colours. Spherical aberration is not removed completely. But crosswire can be connected.

Simple Microscope or Magnifier

$$\text{Magnification } M = \left(1 + \frac{D}{f}\right)$$

Compound Microscope

$$\text{Magnification } M = \frac{v_0}{u_0}\left(1 + \frac{D}{f}\right) = \frac{L}{f_0} \cdot \frac{D}{f_e}$$

for normal adjustment where L is length of the microscope tube.

$M = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)$ for least distance vision.

Length of microscope tube or separation between two lenses $L = v_0 + u_e$.

Resolving Power of Microscope

R.P. $= \frac{\mu \sin \mu}{0.61\,\lambda}$ for self luminous points.

R.P. $= \frac{2\mu \sin \mu}{\lambda}$ for non luminous points.

N.B. Resolving power can be increased if we immerse the objective in an oil and use UV light.

However, resolving power of electron microscope is maximum. Magnification is as high as 80,000. Limit of resolution $= \frac{1}{\text{R.P.}}$

Telescope (Astronomical) is of the types:

(a) Reflecting

(b) Refracting

(c) Radio telescope

Reflecting type is made with concave mirror. Focal length of concave mirror > 1 m (objective).

In refracting type telescope, objective has large focal length and large aperture $f \geq 1$ m, aperture ≥ 2 inches.

Magnification (Normal setting) $M_N = \frac{f_o}{f_e}$

and $L = f_o = f_e$.

Least distance of distinct vision setting

$$M_{LD} = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$$

and $L = f_o + u_e$

Resolving power of telescope R.P. $= \frac{\alpha}{1.22\lambda}$ where α is aperture.

Terrestrial Telescope

Magnification (Normal setting) $M_N = \frac{f_o}{f_e}$

and $L = f_o + 4f_{er} + f_e$ where f_{er} is focal length of erecting lens.

Least distance setting

$M_{LD} = \frac{-f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$ and $L = f_o + 4f_{er} + u_e$.

Rayleigh's Scattering

$\propto \frac{1}{\lambda^4}$. That is why sky appears blue. Rising and setting sun appear red and danger signals are red in colour.

Fig.

Fig. 3.22

Rayleigh's Criterion for just Resolution

Two light sources close together are said to be just resolved if minima of one falls on the maximum of other as shown in Fig. 3.22(a).

WAVE OPTICS

Concepts and Equations

Interference

If light waves emitted form two coherent sources superpose then it results in variation of intensity with distance. At certain places intensity is maximum and at other places intensity is minimum. This phenomenon is called interference.

Coherent Sources

Two sources/wave trains are said to be coherent if there is a constant zero phase difference between them. No two different sources (except lasers) could be coherent. Coherent sources are to be derived from a single source. Their state of polarisation remains same. Laser is considered highly coherent.

If coherent sources have phase shift ϕ then $\phi \neq f(t)$ and $\frac{d\phi}{dt} = 0$. Coherent sources can be obtained by division of wave front or by division of amplitude.

Coherent source obtained with

division of wave front	division of amplitude
(i) YDSE (young's double slit experiment)	(i) Newton's rings
(ii) Fresnel biprism	(ii) Thin films
(iii) Lloyd's mirror	(iii) Interferometer

Wave front is the locus of all adjacent parts at which the phase of vibration of a physical quantity associated with the wave is the same. That is, at any instant, all points on a wave front are at the same part of the cycle of their vibration. Wave fronts in general may be of three types:

(a) Spherical
(b) Cylindrical
(c) Plane or Planar

Spherical wave fronts are generated form a point source or circular slit.

Cylindrical wave front results from a line source or rectangular slit.

Plane wave front is either of the two if the source is at infinity.

Fig.

Fig. 3.23 (a) Wave front (Spherical)

Fig.

Fig. 3.23(b) Illustrated of plane wave front

Constructive interference occurs when the coherent waves superpose in phase or the path difference is integral multiple of the wavelength or even multiple of half the wavelength. This type of "interference" is also called "reinforcement" as light intensity increases, i.e., bright fringes are formed. We may call such points or curves as antinodal. See Fig. 3.24(a). Destructive interference occurs when the coherent waves superpose out of phase or path difference is an odd multiple of half the wavelength. Dark fringes are formed. We may call such points or curves as nodal as illustrated in Fig. 3.24(b)

Fig.

Fig. 3.24(a) Constructive Interference

Fig.

Fig. 3.24(b) Destructive Interference

Path differences $\Delta x = n\lambda$ for constructive interference.

Path difference $\Delta x = (2n + 1)\,\frac{\lambda}{2}$ for destructive interference.

$$\frac{I_{bright}}{I_{dark}} = \frac{I_{max}}{I_{min}}$$

$$= \left(\frac{y_{01} + y_{02}}{y_{01} - y_{02}}\right)^2 = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2$$

Condition to obtain sustained interference.

Necessary Condition

The two sources emitting waves must be coherent.

Desirable Conditions

(i) Sources should be monochromatic having same frequency.

(ii) They shall have same amplitude.

(iii) They shall emit light continuously

(iv) The separation between the two sources shall be small.

In YDSE

Fringe width $\beta = \frac{\lambda D}{d}$ (Difference between two successive dark or bright fringes, i.e., $\beta = x_n - x_{n-1} = \frac{\lambda D}{d}$.

Fig.

Fig. 3.25 Fringe pattern in YDSE

$x_n = \frac{n\lambda D}{d}$ for nth bright fringe

$$x_n = \frac{(2n-1)\lambda D}{2d}$$ for nth dark fringe.

Fig.

Fig. 3.26 Angular Fringe Width

Angular fringe width

$$\theta = \frac{\lambda}{d} = \frac{\beta}{D} \text{ (in radian)}$$

$$= \frac{\lambda}{d} \times \frac{180}{\pi} \text{ (in degrees)}$$

Fringe visibility

$$= \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{\sqrt{2I_1I_2}}{I_1 + I_2}$$

Intensity at any Point

$$I = 2y_0^2 (1 + \cos\delta) = 4I' \cos^2\left(\frac{\delta}{2}\right).$$

Assuming both sources emit waves of equal amplitude y_0 or equal intensity I'. δ is phase shift between two superposing waves.

$$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\delta$$

$$= (y_{01}^2 + y_{02}^2 + 2y_{01}y_{02}\cos\delta)$$

If intensities or amplitude of superposing waves are unequal.

If YDSE is immersed in a liquid of refractive index μ then fringes shrink and hence fringe pattern shrinks.

$$\beta_{new} = \frac{\beta}{\mu} = \frac{\lambda D}{\mu d}$$

or $$x_{n(new)} = \frac{x_n}{\mu} = \frac{n\lambda D}{\mu d}$$ for nth bright fringe.

If a thin slice of thickness t and refractive index μ is inserted in front of one of the slits in YDSE, then central fringe shifts to a position where originally nth fringe was formed such that $(\mu - 1)t = n\lambda$ or $\Delta x = \frac{D(\mu-1)t}{d}$.

In Fresnel biprosm both the sources S_1 and S_2 are virtual as shown in Fig.3.5

$$D = a + b$$

$$d = 2a\,\delta = 2a(\mu - 1)\,\alpha$$

where α is angle of biprism

$$\beta = \frac{\lambda D}{d} = \frac{\lambda(a+b)}{2a(\mu-1)\alpha}$$

$$x_n = \frac{n\lambda D}{d} = \frac{n\lambda(a+b)}{2a(\mu-1)\alpha}$$ for nth bright fringe.

$$x_n = \; = \frac{(2n-1)\,\lambda(a+b)}{4\alpha(\mu-1)\alpha}$$ for nth dark fringe.

Fig.

Fig. 3.27 Fringe pattern in Fresnel biprism

If displacement method is used then $d = \sqrt{d_1 d_2}$.

If Fresnel biprism is immersed in a liquid of refractive index μ', then

$$\beta_{new} = \frac{\lambda}{\mu'}(a+b),2a\left(\frac{\mu}{\mu'} - 1\right)\alpha$$

$$= \frac{\lambda(a+b)}{2a(\mu - \mu')\alpha}$$

In Lloyd's Mirror

Condition of nth bright and dark fringe obtained in Lloyd's mirror gets reversed to what was obtained in YDSE; because of reflection an additional phase shift of π or an additional path difference $\frac{\lambda}{2}$ is achieved.

That is, $x_n = \frac{n\lambda D}{d}$ for nth dark fringe and $x_n = \frac{(2n-1)\lambda D}{2d}$ for nth bright fringe.

In Lloyd's mirror one of the sources is real and other is virtual or image source.

Path difference = $2\mu t \cos r$

$= (2n+1)\frac{\lambda}{2}$ for nth bright fringe and $2\mu t \cos r = n\lambda$ for nth dark fringe. In reflected light

$$\left.\begin{array}{l}\text{Path difference } 2\mu t \cos r = n\lambda \\ 2\mu t \cos r = (2n+1)\frac{\lambda}{2}\end{array}\right\} \text{for refracted or transmitted light}$$

Wedge Shaped Film

Fringe Width $\beta = \frac{\lambda}{2\theta}$, since

$\theta = \frac{1}{x_n}$, Therefore $\beta = \frac{\lambda x_n}{2t}$

If plates are kept in a liquid of refractive index μ

$\beta = \frac{\lambda}{2\mu\theta} = \frac{\lambda x_n}{2\mu t}$ or $2\mu t = n\lambda$

$t_{min} = \frac{\lambda}{2}$. It is due to interference that a soap bubble appears bright colour or oil drops spilled on road in rainy seasons appear of brilliant hue.

Fig.

Fig. 3.28 Wedge shaped film

Time of coherence (t_i) is the time during which electric field vector is in the sinusoidal form. Its value is 10^{-10} s.

Coherence Length $L_C = ct_c$. Note that if path difference > L_C coherence nature is lost. Therefore we cannot keep distance between two slits or sources > 3 cm.

Diffraction

The bending of wave from the obstacles of size of the order of wavelength is termed as diffraction. Planar or plane wave front is required for diffraction to take place. Diffraction is of two types (a) Freshnel Class of diffraction (b) Fraunhoffer class of diffraction.

Table 3.3

Fresnel Class	Fraunhoffer Class
1. The source is at a finite distance.	The source is at infinite distance.
2. No optical aid is required.	Optical aid in the form of collimating lens and focusing lens are required.
3. Frings are not sharp and well defined.	Fringes are sharp and well defined.

Table 3.4

Interference	Diffraction
1. Fringes are formed due to superposition of wave trains emitted from two coherent sources.	Fringes are formed due to superposition of bent rays due to superposition of secondary wavelets.
2. Intensity of each fringe is equal.	Intensity falls as the fringe order increases.
3. Number of fringes is and quite large.	Number of fringes is finite (small)
4. Fringe width is equal for each fringe.	Fringe width of primary and secondary maxima are different.

Huygen's Principle

1. Each point on the primary wave front is a source of secondary wavelets.
2. Secondary wavelets move only in forward direction.
3. Secondary wavelets can superpose to produce disturbances.

Secondary wavelets as well as primary wavefronts move with c (speed of light).

Diffraction from a single slit

Fig.

Fig. 3.29 Single slit diffraction

Path difference = BC = AB sin θ

= d sin θ

For minima

d sin θ = nλ

$\sin\theta = \frac{x_n}{D}$. Thus $\frac{dx_n}{D} = n\lambda$

$x_n = \frac{n\lambda D}{d}$ for nth minima.

Note D = f (of focussing lens)

Fringe with $\beta_{primary} = \frac{2\lambda D}{d}$

and $\beta_{secondary} = \frac{\lambda D}{D}$ (fringe width for secondary maxima is half the primary maxima).

Angular fringe width $\beta_{primary} = \frac{2\lambda}{d}$ (radian)

$= \frac{2\lambda}{d} \times \frac{180}{\pi}$ (degrees)

Angular fringe width $\beta_{secondary} = \frac{\lambda}{d}$ (radian)

$= \frac{\lambda}{d} \times \frac{180}{\pi}$ (degree)

If $\beta = \frac{\pi\, d \sin\theta}{\lambda}$ then $I = \frac{I_0 \sin^2\beta}{\beta^2}$

If aperture is circular then $\sin\theta = \frac{1.22\lambda}{r}$ where r is radius of aperture.

Radius of first dark ring $R = \frac{1.22\lambda D}{r}$ $= \frac{1.22\lambda f}{r}$.

Polarisation

If plane of vibration is fixed then light will travel in a single direction. Such a state is called "plane polarised light."

In the fig. 3.30 electric field varies along y-axis and magnetic field along z-axis, wave travels x-axis, plane of polarisation is y – z.

If $E_v = E_o \sin(\omega x - kx)$ is the electric field along y-axis and $B_z - B_o \sin(\omega t - kx)$ is the magnetic field along z-axis then wave progresses in x-direction.

Only transverse waves can be polarised, longitudinal waves cannot be polarised. Plane polarised light can be achieved using

(a) Reflection

(b) Refraction

(c) Scattering

(d) Nicol prism

(e) Birefracting crystals

Fig.

Fig. 3.30 Plane polarised light

Brewester's Law

If light is incident on the interface of two media such that the angle between reflected and refracted radiations is 90° then reflected rays are completely polarised. Angle of incidence is called "angle of polarization" (θ_p).

Then $\mu = \tan \theta_p$

Malus Law

When the plane of polarisation is rotated by an angle θ then intensity of emergent light is given by $I = I_o \cos^2 \theta$. I_o is intensity of incident polarised light. In birefracting analysis there are two rays – ordinary and extraordinary. The extraordinary ray does not follow law of refraction. If the velocity of extraordinary ray is greater than that of ordinary ray such crystals are called negative crystals. Examples of negative crystal are Iceland spar, foruma line, sappire, ruby, emerald and appetite. If the ordinary ray has higher velocity than such crystals are called positive crystals. Examples of positive crystals are quartz, iron oxide.

If the amplitude of two waves are unequal and angle between the two is $\frac{\pi}{2}$ or path difference is $\frac{\pi}{4}$ then an elliptically polarised wave front results, it could be elliptically.

SPECTRUM OF LIGHT AND PHOTOMETRY

Concepts and equations

Spectrum

A collection of dispersed light giving its wavelength composition is called a "spectrum." For example, hydrogen spectrum has Lyman series, Balmer series, Paschen series, Brackett series, P-fund series etc. and when a white light is incident on a prism a spectrum of different colours form red to violet is observed.

Pure and Impure Spectrum

If each colour gives its sharp impression in the spectrum, then a well defined line spectrum is obtained. Such a spectrum is called pure spectrum. To achieve pure spectrum — (i) The beam of light incident on the dispersing element (prism or diffraction grating) should be parallel or collimated. (ii) The dispersed light should be focussed in such a way that all the rays of a particular wavelength are collected at a place.

N.B. A spectrum will satisfy the above requirements.

If the slit is wide, different points of the slit produce separate spectra which overlap each other. Thus colour impression gets diffused due to overlap resulting into an impure spectrum.

Kinds of Spectra

Broadly speaking we can divide the spectrum into two types — emission spectrum and absorption spectrum.

Light is emitted by an object when it is suitably excited by heating or by passing an electric discharge etc. If this light is passed through a dispersing element, emission spectrum is obtained. A lot of information about the source material can be obtained from the emission spectrum. Emission spectrum may be of three types:

Continuous Spectrum

If the source is a hot solid such as bulb filament or liquid, the spectrum is continuous. Light emitted by a bulb, candle or red hot iron has continuously varying wavelengths. Even X-ray spectrum is continuous.

Line Spectrum

When substances in its atomic state (gaseous or vapour state) de-excite, they produce bright colour lines. For example when common salt is thrown in a campfire, only a few colours appear in the form of isolated sharp parallel lines. Each lien is the image of spectrograph slit deviated through an angle that depends upon the wavelength. A spectrum of this sort is called a line spectrum. For example sodium gives D_1 and D_2 doublet (589 and 589.6 nm). Hydrogen spectrum is well studied and so on.

Band Spectrum

The molecular energy levels are generally grouped into several bunches, each bunch widely separated from the other but levels in a bunch are close to each other. The wavelengths emitted by such molecules are also grouped. Each group retains its identity (is separated from the other). The wavelengths in a group being close to each other and appear as continuous. The spectrum looks like a band of colours.

Absorption Spectrum

When white light having all the wavelengths is passed through an absorbing material, the material may absorb certain wavelengths selectively (to get excited). These wavelengths will disappear when the transmitted light is dispersed (passed through a prism or grating). Dark lines or bands at the missing wavelengths appear on an otherwise bright continuous coloured background. Such a spectrum is called absorption spectrum. It is of two types line absorption spectrum and band absorption spectrum. When sunlight is dispersed certain sharply defined dark lines are seen. These lines are called Fraunhoffer lines. Fig. 3.31(a) and Fig. 3.31(b) illustrates emission and absorption process.

Fig.

Fig. 3.31

Speed of Light using Fizeau Method

$c = \frac{2Dn\omega}{\pi} = 4\,Dnf$ where D is distance from the rotating wheel of the mirror ω is angular speed of rotation of the wheel when image is completely unseen for the first time and n is number of teeth in the wheel or number of rotations per second, $\omega = 2\pi\,f$ where f is linear frequency.

Foucault's Method to find Speed of Light

$c = \frac{4R^2\omega a}{S(R + b)}$ where R is the radius of concave mirror, a is distance between lens and source, b is distance between plane mirror and lens and S is shift in image.

Michelson Method to find Speed of Light

$c = \frac{DN\omega}{2\pi} = DfN$ where N is number of faces in a polygonal mirror, ω is angular speed of rotation and D is the distance travelled by light on reflections from polygonal mirror. $f = \frac{\omega}{2\pi}$ is the linear frequency.

Fresnel Distance

$Z_f = \frac{a^2}{\lambda}$ where a is slit width. Z_f describes the distance travelled by a beam without appreciable broadening of the beam.

Lambert's Cosine Law

The surfaces which radiate according to the Lambert's Cosine Law are called perfectly diffused. $I = I_\theta \cos\theta$.

Luminous Flux

Radiation emitted by a source has components corresponding to a wide range of wavelengths. Different component wavelengths have different energies and different brightness. The luminous flux is a quantity directly representing the total brightness producing capacity of the source. Its unit is lumen. Luminous flux of a source of $\frac{1}{685}$ W emitting monochromatic light of wavelength 555 nm is called 1 lumen. That is, a 1 watt source emitting a monochromatic light of wavelength 555 nm emits 685 lumen.

$$\frac{\text{Luminous flux of a source of a given wavelength}}{\text{Luminous flux of a 555 nm source of same power}}$$

Luminous efficiency

$$= \frac{\text{Total luminous flux}}{\text{Total radiant flux}}$$

$$= \frac{\text{Luminous flux emitted}}{\text{Power input to the source}}$$

Luminous Intensity or Illuminating Power (I)

Luminous flux per unit solid angle is defined as luminous intensity. Its unit is candela (cd).

$I = \frac{dF}{d\Omega} = \frac{F}{4\pi}$ where F is luminous flux and Ω is solid angle.

1 Candela is the luminous intensity of a black body of surface area $\frac{1}{60}$ cm^2 placed at the freezing temperature of platinum at a pressure of 101.325 N m^{-2}.

Illuminance (E) is the luminous flux incident per unit area $\frac{dF}{dA}$ units lumen m^{-2} or Lux. CGS unit is Phot.

Law of Photometry

A photometer is used to compare intensities of two sources $\frac{I_1}{I_2} = \left(\frac{d_1}{d_2}\right)^2$ where d_1 and d_2 are distances of the source from photometer.

MCQs

1. What is optics?
   - (a) the scientific study of light and vision
   - (b) the scientific study of sound
   - (c) the scientific study of time
   - (d) the scientific study of fluids

2. Which of the following forms of electromagnetic (EM) energy has the longest wavelength?
   - (a) microwaves
   - (b) radiowaves
   - (c) infrared waves
   - (d) visible light

3. Which of the following form of EM energy has the highest frequency?
   - (a) x-rays
   - (b) gamma rays
   - (c) ultraviolet waves
   - (d) infra-red waves

4. In comparison to the other forms of EM radiation, radiowaves have the
   - (a) highest energy and highest frequency
   - (b) lowest energy and lowest frequency
   - (c) lowest energy and highest frequency
   - (d) highest energy and lowest frequency

5. Which form of electromagnetic radiation is used in RADAR?
   - (a) long wavelength ultraviolet waves
   - (b) short wavelength microwaves
   - (c) short wavelength infrared waves
   - (d) long wavelength radio waves

6. Infrared radiation is also known as
   - (a) radio signals
   - (b) heat radiation
   - (c) magnetic resonance waves
   - (d) RADAR

7. Which scientist made the first successful terrestrial measurement of the speed of light?
   - (a) Isaac Newton
   - (b) Ole Romer
   - (c) Armand Fizeace
   - (d) Albert Michelson

8. Which branch of medicine deals with the anatomy, physiology and diseases of the eye?
   - (a) Ophthalmology
   - (b) Cardiology
   - (c) Radiology
   - (d) Andrology

9. Which law states that the angle of incidence equals the angle of reflection?
   - (a) law of refraction
   - (b) law of reflection
   - (c) Snell's, law
   - (d) none of the above

10. Which Muslim scientist gave the first clear description and correct analysis of pinhole camera in the Book of Optics (Kitab al-Manazir) in 1021 AD?
    - (a) Ibn al-Haytham (Alhazen)
    - (b) Ibn Ishaq al-Kindi (Alkindus)
    - (c) Nasir al-Din al-Tusi
    - (d) Ibn Musa al-Khwarizmi

11. Which Muslim scientist is regarded as "father of optics"?
    - (a) Ibn al-Haytham
    - (b) Ibn Ishaq al-Kindi

(c) Al-Khawarizmi
(d) Nasir al-Din al-Tusi

12. Which of the following is used for reducing spherical aberrations in optical instruments?
    (a) plano-convex lens
    (b) concave lens
    (c) spherical mirrors
    (d) plane mirrors

13. Which of the following is used for the failure of a lens to form a sharp and distinct image?
    (a) distortion
    (b) astigmation
    (c) chromatic aberration
    (d) spherical aberration

14. Power of lens is measured in
    (a) cm (b) cm^{-1}
    (c) dioptress (d) meters

15. A double convex air bubble in water acts as
    (a) converging lens
    (b) diverging lens
    (c) plane slab
    (d) none of the above

16. The blurring of the image due to dispersion in lens is called
    (a) spherical aberration
    (b) chromatic aberration
    (c) astigmation
    (d) curvature of image field

17. Electromagnetic wave theory was proposed by
    (a) Maxwell (b) Hertz
    (c) Fizeace (d) Huygen

18. Which of the following properties exhibits that light waves are transverse in nature?
    (a) interference (b) diffraction
    (c) polarization (d) refraction

19. Electromagnetic waves transport
    (a) energy only
    (b) moment only
    (c) energy and momentum
    (d) none of the above

20. Huygen's wave theory fails to explain
    (a) diffraction
    (b) polarization
    (c) interference
    (d) refection

21. The light speed is vacuum depends on
    (a) frequency
    (b) amplitude
    (c) wavelength
    (d) none of the above

22. The minimum angle of incidence for which total internal reflection can occur is called the
    (a) critical angle
    (b) acute angle
    (c) obtuse angle
    (d) right angle

23. Which of the following are electromagnetic waves?
    (a) x-rays
    (b) microwaves
    (c) light
    (d) all of the above

24. What are SI units of Plank's constant?
    (a) J.s. (b) W.s.
    (c) N.s. (d) J.m.

25. Which law states that the ratio of the angles θ_a and θ_b is equal to the inverse ratio of the two indexes of refraction?
    (a) Snell's law
    (b) Ohm's law
    (c) Newton's law
    (d) Kirchoff's law

26. The dependence of wave speed and index of refraction on wavelength is called
    (a) refraction
    (b) dispersion
    (c) reflection
    (d) echo

27. The band of colours is called
    (a) spectrum
    (b) prism
    (c) medley
    (d) LASER

28. Who set forwarded the corpuscular theory of light?
    (a) Newton
    (b) Huygen
    (c) Hertz
    (d) Einstein

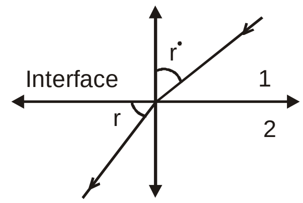
29. Newton's corpuscular theory failure to explain which phenomenon of light?
    (a) diffraction
    (b) interference
    (c) polarization
    (d) all of the above

30. In water drops, rainbows are formed by
    (a) refraction
    (b) reflection
    (c) dispersion
    (d) all of the above

31. Rebounding of light from a polished surface like a mirror is termed as
    (a) reflection
    (b) refraction
    (c) dispersion
    (d) interference

32. Reflection from a rough surface (such as a wall) is called
    (a) refraction (b) dispersion
    (c) diffusion (d) interference

33. When an oblique ray of light enters from one medium to another then it changes its path. Such a phenomenon is called



    (a) reflection (b) refraction
    (c) dispersion (d) diffusion

34. Splitting of complex light into its constituent colours is called
    (a) reflection (b) refraction
    (c) dispersion (d) diffusion

35. Which of the following are defects in human eye?
    (a) Myopia (short sightedness)
    (b) Hypermetropia (long sightedness)
    (c) Presbyopia
    (d) all of the above

36. Which law states that when a perfect polarizer is placed in a polarized light beam, the intensity I, of the light passes through is given by $I = I_0 \cos^2 \phi_0$?
    (a) Mauls' law
    (b) Hooke's law
    (c) Bragg's law
    (d) Dalton's law

37. Which law states that the maximum polarization of a light ray may be achieved by letting the ray fall on a surface of a transparent medium such that the refracted ray makes an angle of 90° with the reflected ray?
    (a) Mauls' law
    (b) Bragg's law
    (c) Brewster's law
    (d) Lenz's law

38. What would be the colour of sky throughout the day, if the earth had no atmosphere?
    (a) blue (b) black
    (c) red (d) white

39. When we look at the day time sky, the light that we see is sunlight that has been absorbed and then re-radiated in different directions. This process is called ________
    (a) scattering (b) diffusion
    (c) mirage (d) rainbow

40. Clouds are white because they efficiently scatter sunlight of all ________.
    (a) colours
    (b) wavelengths
    (c) frequencies
    (d) phases

41. Which principle assumes that every point of a wavefront may be considered the source of secondary wavelets that spread out in all directions with a speed equal to the speed of propagation of the wave?
    (a) Huggen's principle
    (b) Archimedes' principle
    (c) Relativity principle
    (d) Landauer's principle

42. Huygens' concept of secondary wave is used to
    (a) determine the velocity of light
    (b) locate the wavefront
    (c) explain polarization
    (d) find the power of a lens

43. What is defined as the ratio of image height to the object height?
    (a) lateral magnification
    (b) angular magnification
    (c) both of the above
    (d) none of the above

44. Which law gives the angles for coherent and incoherent scattering form a crystal lattice?
    (a) Mauls' law
    (b) Bragg's law
    (c) Brewster's law
    (d) Hooke's law

45. Which instrument measures the magnification of a telescope?
    (a) Lactometer
    (b) Dynamometer
    (c) Wattmeter
    (d) Ammeter

46. Which of the following are defects in lenses?
    (a) chromatic aberration
    (b) spherical aberration
    (c) astigmatism
    (d) all of the above

47. Which principle states that the path taken by a ray of light between any two points in a system is always the path that takes the least time (or the shortest optical path)?
    (a) Fermat's principle (principle of least time)
    (b) Huygens' principle
    (c) Archimedes' principle
    (d) Landauer's principle

48. Which term is used for human eye defect "near sightedness"?
    (a) Myopia
    (b) Presbyopia
    (c) Hypermetropia
    (d) Cataract

49. Which of the following are optical telescopes?
    (a) refracting telescope (reflectors)
    (b) refracting telescopes (refractors)
    (c) catadioptric telescopes
    (d) all of the above

50. Which mirror be used for obtaining a parallel beam of light from a small lamp?
    (a) plane mirror
    (b) convex mirror
    (c) concave mirror
    (d) any one of the above

51. Which property of light waves does not vary with the medium?
    (a) frequency
    (b) velocity
    (c) wavelength
    (d) amplitude

51. Which is the complementary colour of orange?
    (a) blue (b) yellow
    (c) violet (d) indigo

53. Electromagnetic waves are produced by
    (a) charge at rest
    (b) accelerated charge
    (c) heating a conductor
    (d) none of the above

54. Wavelength of a LASER can be used as a standard of
    (a) angle
    (b) time
    (c) length
    (d) temperature

55. Which of the following waves cannot be polarised?
    (a) x-ryas
    (b) radio waves
    (c) ultraviolet rays
    (d) sound waves

56. Dispersive power of a prism depends upon the wavelength of the light used and is
    (a) more for large wavelengths
    (b) less for large wavelengths
    (c) more for small wavelengths
    (d) less for small wavelengths

57. What determines the polarization of an electromagnetic wave?
    (a) the magnetic field only
    (b) the electric field only
    (c) both the electric and magnetic fields only
    (d) none of the above

58. Two lenses of focal lengths f_1 and f_2 are separated by a distance d. What is the condition that the condition be achromatic?
    (a) $f_1 + f_2 = 2d$
    (b) $f_1 - f_2 = 2d$
    (c) $f_1 f_2 = d$
    (d) $f_1 f_2 = 2d$

59. The inability of rays of different colours to converge a single point after refraction through a convex lens a called
    (a) coma
    (b) spherical aberration
    (c) distortion
    (d) chromatic aberration

60. The power of lens in dioptress (D) is
    (a) its focal length in meters
    (b) the reciprocal of its focal length in meters'
    (c) the reciprocal of length in meters
    (d) the focal length in centimeters

61. What is a "zoom lens"?
    (a) a lens having a fixed focal length
    (b) a lens having variable focal length
    (c) a lens used in telescopes
    (d) none of these

62. Two convex lenses of focal length f used in combination become telescopic when the distance between them is
    (a) f (b) 2f
    (c) 4f (d) 4/2

63. If D_1 and D_2 are the power of two lenses placed in contact, then the power of the combination will be
(a) D_1D_2 (b) D_1/D_2
(c) $D_1 - D_2$ (d) $D_1 + D_2$

64. The variation of focal length of a lens when we pass from the central portion to periphery is called
(a) coma
(b) astigmatism
(c) spherical aberration
(d) chromatic aberration

65. The defect in image due to oblique centric rays falling on the lens is called
(a) coma
(b) spherical aberration
(c) astigmatism
(d) curvature of image field

66. Which of the following defects is removed by Huygen's eye-piece?
(a) astigmatism
(b) chromatic aberration
(c) spherical aberration
(d) both a and b

67. On which property of lens, longitudinal chromatic aberration depends upon?
(a) resolving power
(b) dispersive power
(c) magnifying power
(d) none of these

68. Which of the following be used for reducing mechanical aberration in optical instruments?
(a) plane mirrors
(b) spherical mirrors
(c) concave lenses
(d) plano-convex lenses

69. The aberration in the image formed by a lens due to different wavelengths present in a source is called
(a) spherical aberration
(b) chromatic aberration
(c) achromatic aberration
(d) none of these

70. The ability of a convex lens to produce convergence in a parallel beam is called its
(a) magnification
(b) focal length
(c) power
(d) strength

71. Image formed by a concave lens is
(a) real
(b) magnified
(c) virtual
(d) none of the above

72. Two thin lenses in contact, produce a combined power of +10 dioptress. When they are 0.25 m apart the power reduced to +6 dioptress. The power of the lens in dioptress are
(a) 1 and 9 (b) scach
(c) 4 and 6 (d) 2 and 8

73. On which of the following the object size as perceived by eye depends upon?
(a) actual size of the object
(b) aperture of the pupil
(c) object distance from the eye
(d) size of the image formed on the retina

74. Why an eye is not able to see objects closer than 25 cm?
(a) focal length of the eye is 25 cm
(b) distance of retina from eye-lens is 25 cm
(c) eye is not able to decrease the focal length beyond a limit
(d) none of these

75. What is the focal length of a normal eye-lens?
(a) 1 mm (b) 2 cm
(c) 25 cm (d) 1 m

76. The distance of the eye-lens form the retina is x. For a normal eye, the maximum focal length the eye-lens is
(a) = x (b) < x
(c) > x (d) = 2x

77. The intensity produced by a long cylindrical light source at a small distance r from the source is proportional to
(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$
(c) $\frac{1}{r^3}$ (d) none of these

78. As the wavelength is increased from violet to red, the luminosity
(a) continuously increases
(b) continuously decreases
(c) increases then decreases
(d) decreases then increases

79. Why danger signals are made red?
(a) our eyes are more sensitive to red colour
(b)the red colour has minimum scattering
(c) the red colour has maximum scattering
(d) none of the above

80. What is the cause of mirage in desert areas?
(a) the refractive index of atmosphere increases with height
(b) the refractive index of atmosphere decreases with height
(c) the refractive index of atmosphere remains constant
(d) scattering

81. Why a glass plate inside a colourless liquid become invisible?
(a) the colours of both are same
(b) liquid with glass surface
(c) the densities of both are same
(d) their refractive indices are same

82. What is the relation between energy E and momentum p of a photon?
(a) $E = pc$ (b) $E = \frac{p}{c}$
(c) $p = Ec$ (d) $E = \frac{p^2}{c}$

83. What is the total energy density of an electromagnetic wave in vacuum?
(a) $e_0 \frac{E^2}{3}$ (b) $e_0 E^2$
(c) $\frac{\epsilon_0 E^2}{2}$ (d) $\frac{E^2}{\epsilon_0}$

84. All particles of a wavefront vibrate
(a) in same phase
(b) in opposite phase
(c) up and down
(d) left and right

85. What is the unit of poynting vector?
(a) watt (b) joule
(c) $\frac{\text{watt}}{\text{m}^2}$ (d) $\frac{\text{joule}}{\text{m}^2}$

86. When a light ray enters form air into water then its wavelength
(a) increases
(b) decreases
(c) becomes infinity
(d) remains constant

87. Which phenomenon verifies the transverse nature of light waves?
(a) reflection
(b) refraction
(c) polarization
(d) interference

88. The spin of photon is
    (a) h (b) $\frac{h}{2}$
    (c) 2h (d) $\frac{h}{3}$

89. What would be the colour of sky in the absence of atmosphere?
    (a) blue (b) indigo
    (c) red (d) black

90. For total internal reflection, the light ray enters
    (a) form rearer to denser medium
    (b) from denser to rearer medium
    (c) medium of same refractive index
    (d) none of the above

91. Why a diamond shines so brightly? Due to
    (a) scattering of light
    (b) refraction of light
    (c) dispersion of light
    (d) total internal reflection of light

92. Which of the following colours scatters minimum?
    (a) blue (b) violet
    (c) yellow (d) red

93. The sun appears elliptical before sunset due to
    (a) reflection
    (b) refraction
    (c) scattering
    (d) total internal reflection

94. Sunlight can undergo internal reflection if it enters from
    (a) glass to air
    (b) air to glass
    (c) air to water
    (d) water to glass

95. The refractive index of diamond is 2. What will be the velocity (in cm/sec) in diamond?
    (a) 3×10^{10}
    (b) 2×10^{10}
    (c) 1.5×10^{10}
    (d) 6×10^{10}

96. A pencil dipped partially into water appears bent because of
    (a) reflection of water surface
    (b) diffraction at water surface
    (c) refraction at water surface
    (d) water is a fluid

97. A red flower when viewed through blue light, appear
    (a) red (b) black
    (c) blue (d) violet

98. Which of the following is a correct statement?
    (a) light exhibits wave nature in propagation and particle nature in mutual interaction with matter
    (b) wave theory is valid for long wavelength region and quantum theory is valid for short wavelength region
    (c) the main cause of microwaves being unfit for vision is the particle nature of EM waves
    (d) all of the above

99. Which of the following are examples of images?
    (a) your reflection in the bathroom mirror
    (b) view of the moon through a telescope
    (c) the patterns seen in kaleidoscope
    (d) all of the above

100. The Ramsden eyepiece consist of
    (a) two plano-convex lens with same focal length
    (b) two sets of doublets
    (c) an achromatic doublet
    (d) none of the above

101. Plossl or symmetrical eye-piece consists of
   (a) two plano-convex lens with same focal length
   (b) two sets of doublets
   (c) an achromatic doublet
   (d) none of these

102. Kellner or Achromat eye-piece consists of
   (a) two plano-convex lens with same focal length
   (b) two sets of doublets
   (c) an achromatic doublet
   (d) none of these

103. Photometers are used to measure
   (a) illuminance
   (b) irradiance
   (c) fluorescence
   (d) all of the above

104. Which of the following factors determines the resolving power of an instrument?
   (a) magnification
   (b) focal length of objective
   (c) diameter of objective
   (d) none of the above

105. A biprism consists of
   (a) two parallel glass plates
   (b) two acute angled prisms
   (c) two obtuse angled prisms
   (d) none of the above

106. The branch of optics deals with the nature and propagation of light is called
   (a) geometric optics
   (b) physical optics
   (c) quantum optics
   (d) none of these

107. When the frequency of an EM wave and ultrasonic wave are same then
   (a) their wavelengths should be same
   (b) wavelength of EM wave will be more
   (c) wavelength of ultrasonic wave will be more
   (d) none of these

108. Which of the following is not EM in nature?
   (a) x-rays
   (b) γ-rays
   (c) cathode rays
   (d) infra-red rays

109. Which of the following is an important property of EM waves?
   (a) electric and magnetic fields are in same phase
   (b) electric and magnetic fields are out of phase
   (c) both the fields are sometimes in phase and sometimes out of phase
   (d) all of the above

110. Which of the following colour of light passes through glass with minimum speed?
   (a) red (b) green
   (c) yellow (d) violet

111. Colour of light is determined by its
   (a) amplitude
   (b) velocity in air
   (c) wavelength
   (d) state of polarization

112. The linear distance between succession points having same phase in a wave disturbance is called
   (a) frequency
   (b) amplitude
   (c) phase difference
   (d) wavelength

113. Which of the following phenomenon can not be explained by Newton's corpuscular theory?
   (a) reflection

(b) interference
(c) refraction
(d) propagation

114. Which parameter is an indication of colour of a star?
(a) weight
(b) distance
(c) size
(d) temperature

115. Which phenomenon is responsible for formation of shadows?
(a) interference of light
(b) diffraction of light
(c) polarization of light
(d) propagation of light

116. What happens in the phenomenon of interference?
(a) annihilation of light
(b) re-distribution of light
(c) local addition of intensity
(d) none of these

117. Monochromatic light passing through a thick prism is
(a) polarised
(b) dispersed
(c) diffracted
(d) deviated

118. Which of the following is used as a remedy for defect of hypermetropia?
(a) convex lens
(b) concave lens
(c) cylindrical lens
(d) bifocal length lens

119. Which lens is used for curing colour blindness?
(a) contact lens
(b) cylindrical lens
(c) bifocal length lens
(d) none of the above

120. The eye's ability to focus near as well as distant objects is termed what?
(a) myopia
(b) persistence of vision
(c) power of accommodation
(d) astigmatism

121. A light ray is reflected from a denser medium. What is phase and path difference?
(a) 0, 0 (b) $\pi, \frac{\lambda}{2}$
(c) π, λ (d) $\frac{\pi}{2}, \frac{\lambda}{2}$

122. Least distance of distinct vision
(a) decreases with increase in age of a person
(b) increases with increase in age of a person
(c) varies with the age of a person
(d) does not vary with age of a person

123. Short sightedness in the eye occurs due to the
(a) contraction of eyeball
(b) increase in focal length of eye-lens
(c) reduction in focal length of eye-lens
(d) reduction in distance between retina and eye-lens

124. A youngman wearing glasses does not require bifocals because he
(a) is farsighted
(b) has the ability to accommodate
(c) is short sighted
(d) does not suffer form coma

125. What is the magnifying power of a convex lens of focal length 5 cm?
(a) 3 (b) 5
(c) 6 (d) 20

126. A rod of refractive index 1.42 is immersed in a liquid of refractive index 1.42. The rod will
   (a) become invisible
   (b) appear slightly bent
   (c) appear slightly raised
   (d) none of the above

127. The critical angle will be maximum when light travels from
   (a) glass to air
   (b) water to air
   (c) glass to water
   (d) water to glass

128. The variation of focal length when we pass from the central position to the periphery is called
   (a) coma
   (b) chromatic aberration
   (c) spherical aberration
   (d) astigmatism

129. Two lenses of focal length of are combined. The resultant focal length is
   (a) f (b) 2f
   (c) $\frac{f}{2}$ (d) zero

130. When a light ray enters a glass slab form air
   (a) its frequency increases
   (b) its wavelength increases
   (c) its wavelength decreases
   (d) neither frequency nor wavelength change

131. Which property of light waves does not vary with the medium?
   (a) velocity
   (b) frequency
   (c) amplitude
   (d) wavelength

132. Which are the types of wavefronts?
   (a) spherical (b) planar
   (c) cylindrical (d) all of the above

133. The waves emitted by a radio transmitter are
   (a) linearly polarised
   (b) unpolarised
   (c) monochromatic
   (d) elliptically

134. Dichorism means selective absorption of
   (a) dispersed light
   (b) scattered light
   (c) unpolarised light
   (d) one of the polarised components

135. To which interference antinodal curves correspond to
   (a) constructive
   (b) destructive
   (c) neither constructive nor destructive
   (d) both constructive and destructive

136. When exposed to sunlight, thin films of oil on water often exhibit brilliant colours due to the phenomenon of
   (a) dispersion
   (b) diffraction
   (c) interference
   (d) acceleration

137. Optically active substances are those which
   (a) produce polarised light
   (b) rotate plane of polarisation of polarised light
   (c) produce double refraction
   (d) none of the above

138. Light transmitted by Nicole prisms is
   (a) unpolarised
   (b) circularly polarised
   (c) plane polarised
   (d) elliptically polarised

139. Which is the light-sensitive tissue in the human eye?
   (a) retina (b) pupil
   (c) iris (d) cornea

140. Which is a thin, circular structure in the eye, containing an aperture with variable diameter? It controls the amount of light reaching the retina
   (a) retina (b) pupil
   (c) iris (d) cornea

141. Eye colour is the colour of
   (a) iris (b) retina
   (c) cornea (d) pupil

142. Which part is a hole located in the centre of the eye that allows light to enter the retina?
   (a) iris (b) pupil
   (c) cornea (d) fovea

143. Which is a transparent front part of the eye that covers the pupil, iris and anterior chamber?
   (a) cornea (b) fovea
   (c) sclera (d) choroid

144. What is the refractive power of cornea in humans?
   (a) 13 dioptress
   (b) 23 aioptress
   (c) 33 disptress
   (d) 43 dioptres

145. Which is a health care profession concerned with eyes as well as vision, visual system and vision information processing in humans?
   (a) optometry
   (b) ophthalmology
   (c) telemetry
   (d) psychology

146. Which is the branch of medicine that deals with the anatomy, physiology and diseases of the eye?
   (a) ophthalmology
   (b) psychology
   (c) andrology
   (d) gynecology

147. Width of the Newton's rings in case of reflection
   (a) increases with the order of the fringe
   (b) decreases with the order o the fringe
   (c) remains unchanged
   (d) none of the above

148. The central ring is bright in case of Newton rings produced by
   (a) reflection
   (b) wedges
   (c) transmission
   (d) none of these

149. The phenomenon of interference is explained by
   (a) complex effect
   (b) Newton's ring
   (c) Raman's spectra
   (d) emission spectra

150. A chromatic fringe can be obtained with
   (a) white light
   (b) coherent light
   (c) incoherent light
   (d) invisible light

151. Which is not an example of interference by division of wavefront
   (a) Lylod's mirror
   (b) Newton's rings
   (c) Fresnel Bi-prism
   (d) Young's slit

152. Which of the following devices can be used to observe interference?
   (a) biprism
   (b) prism
   (c) spectrometer
   (d) photometer

153. Due to which phenomenon soap bubbles shows colour when viewed in white light?
(a) interference
(b) diffraction
(c) scattering
(d) dispersion

154. Soap film exhibit brilliant colours in sunlight due to
(a) dispersion of light
(b) scattering of light
(c) interference of light
(d) diffraction of light

155. Which of the following phenomenon is not an interference phenonmeon?
(a) metallic surface when heated displays colours
(b) soap bubbles in sunlight show colours
(c) sky seems blue at noon but red at dawn and at evening
(d) oil spread on water surface exposed to sunlight

156. A fringe is a path of
(a) constant amplitude
(b) constant phase
(c) same wavelength
(d) none of these

157. Which method produces Newton's rings?
(a) division of wavefront
(b) division of amplitude
(c) addition of amplitude
(d) none of the above

158. Due to which phenomenon, an air bubble in water shines?
(a) dispersion
(b) reflection
(c) diffraction
(d) total internal reflection

159. With which phenomenon, the Compton effect deals?
(a) diffraction (b) interference
(c) scattering (d) polarization

160. Fresnel diffraction is
(a) a far field phenomenon
(b) a near field phenomenon
(c) both of the above
(d) none of the above

161. The spectrum obtained with a grating is called
(a) grating spectrum
(b) impure spectrum
(c) anomalous spectrum
(d) normal spectrum

162. Rising and setting sun appears to be reddish due to
(a) refraction of light rays
(b) scattering of light rays
(c) less temperature at sunset and sunrise
(d) interference of light rays

163. The dispersive power of a grating is
(a) light used
(b) separation of lines
(c) frequency of light used
(d) independent of wavelength

164. In a diffraction pattern, the width of any fringe is
(a) directly proportional to slit width
(b) inversely proportional to slit width
(c) independent of slit width
(d) none of the above

165. With which factor, dispersive power of a grating increases?
(a) order of spectrum
(b) number of lines per centimeter
(c) order and number of lines per centimeter
(d) all of the above

166. What was confirmed by the discovery of polarisation of light?
   (a) transverse nature of light waves
   (b) longitudinal nature of light waves
   (c) light waves are neither waves nor particles
   (d) all of the above

167. Which of the following device produces plane polarised light?
   (a) prism
   (b) bi-prism
   (c) Nicole prism
   (d) none of the above

168. The vibrations of an unpolarised light can take place
   (a) in all planes
   (b) in one plane
   (c) in no plane
   (d) all are false

169. Light waves can be polarised because they
   (a) are transverse in nature
   (b) can be reflected
   (c) have short wavelength
   (d) have high frequencies

170. A plane of polarisation is one in which
   (a) vibrations take place
   (b) no vibrations take place
   (c) longitudinal vibrations take place
   (d) transverse vibrations take place

171. Light produced by a single Nicole is
   (a) unpolarised
   (b) plane polarised
   (c) circulatory polarised
   (d) elliptically polarised

172. Light waves can be polarised because they
   (a) have short wavelengths
   (b) have high frequencies
   (c) can be reflected
   (d) are transverse

173. Which term best describes the nature of light from modern viewpoint?
   (a) waves
   (b) rays
   (c) particles
   (d) photons

174. In photometry what is the total luminous flux incident on the surface, per unit area?
   (a) illuminance
   (b) fluorescence
   (c) luminance
   (d) incandescence

175. What is the emission of light by a substance that has observed light or others electromagnetic radiation of a different wavelength?
   (a) fluorescence
   (b) illuminance
   (c) luminance
   (d) incandescence

176. What is the photometric measure of luminous intensity per unit area of light travelling in a given direction?
   (a) luminance
   (b) illuminance
   (c) fluorescence
   (d) irridiance

177. Candela per square meter (cd/m^2) is the SI unit for
   (a) luminance
   (b) irradiance
   (c) illuminance
   (d) fluorescence

178. In optics, which subfield studies the measurement of electromagnetic radiation, including visible light?
   (a) radiometry
   (b) photometry
   (c) telemetry
   (d) chronometry

179. Which term is used for the emission of visible light from a hot body due to its temperature?
(a) luminance
(b) incandescence
(c) fluorescence
(d) irradiance

180. What is the radiometry term for the power of electromagnetic radiation per unit area at a surface?
(a) fluorescence
(b) luminance
(c) irradiance
(d) incandescence

181. What is the SI unit of irradiance or radiant emittance or radiant existence?
(a) w/m (b) w/m^2
(c) w/m^3 (d) w/m

182. Irradiance due to solar radiation is also called
(a) insulation
(b) isolation
(c) declamation
(d) insolation

183. What is the SI unit of spectral irradiance?
(a) w/m (b) w/m^2
(c) w/m^3 (d) w/cm

184. What is the SI derived unit of illuminnace?
(a) lux or lumens/m^2
(b) candela
(c) phot
(d) candela/cm

185. Which device is used for measuring illuminance in work environments?
(a) luxmeter
(b) photometer
(c) ammeter
(d) wattmeter

186. Which of the following gives discrete emission spectrum?
(a) sun
(b) candle
(c) incandescent filament
(d) mercury vapour lamp

187. For what the wavelength of LASER light can be used as a standard?
(a) time
(b) length
(c) weight
(d) temperature

188. A monochromatic light beam when passed through a prism is
(a) diffracted
(b) deviated
(c) polarised
(d) dispersed

189. Raman effect is due to
(a) coherent scattering
(b) incoherent scattering
(c) no scattering
(d) refraction

190. Which principle does not hold in non linear optics?
(a) Superposition principle
(b) Huygenes' principle
(c) energy conservation principle
(d) none of the above

191. On what principle, interferometers are based on?
(a) diffraction
(b) superposition
(c) interference
(d) scattering

192. Michelson's interferometer can be used to measure
(a) intensity of light
(b) amplitude of disturbance
(c) wavelength of light
(d) none of these

193. The action of a zone plate is similar to a
   (a) concave lens
   (b) convex lens
   (c) glass-plate
   (d) none of these

194. The phenomenon of rotating the plane of vibration of polarised light is called
   (a) polarisation
   (b) optical cavity
   (c) refraction
   (d) reflection

195. Lenses of what diameter are usually not practical?
   (a) less than 1 m
   (b) larger than 1 m
   (c) larger than 5 m
   (d) larger than 10 m

196. Monochromatic light is a single ________
   (a) frequency
   (b) length
   (c) amplitude
   (d) pitch

197. Which type of microscope was the first to be developed?
   (a) optical microscope
   (b) digital microscope
   (c) electron microscope
   (d) none of these

198. Who is generally credited with the invention of first optical microscope?
   (a) Hans Lippershery
   (b) Giovanni Faber
   (c) Galilee Galilei
   (d) Isaac Newton

199. Which of the following structural components microscope is called "Ocular"?
   (a) eyepiece (b) objective
   (c) frame (d) diaphragm

200. Which Dutch scientist improved on a microscope for viewing biological specimens and is called Father of Microbiology?
   (a) Galileo Gulilei
   (b) Robert Hooke
   (c) Auton van Luuwenbhock
   (d) Isaac Newton

201. In which country both microscope and telescope were invented?
   (a) Italy
   (b) England
   (c) India
   (d) Netherlands

202. Which type of telescope works on shorter wavelengths than ultraviolet light?
   (a) ultraviolet telescope
   (b) x-ray telescope
   (c) infrared telescope
   (d) submillimeter telescope

203. Which Italian inventor is credited with describing and sketching the first ideas for contact lens in 1508?
   (a) Galileo Galilei
   (b) Leonardo da Vinci
   (c) Roger Bacon
   (d) Hans Lippershey

204. Which eye problem can be rectified by using divergent lens?
   (a) Myopia (Near-sightedness)
   (b) Hyperopia (For sightedness)
   (c) Presbyopia
   (d) Astigmatism

205. What is the type of corrective lens used to correct or enhance the vision in only one eye?
   (a) axicon
   (b) monocle
   (c) zoom lens
   (d) camera lens

206. Which of the following is a transport optical element with flat, polished surfaces that refract light?
(a) monocle (b) axicon
(c) prism (d) lens

207. Which dimensionless number is a quantitative measure of lens speed?
(a) A-number
(b) F-number
(c) Avogadro's number
(d) none of these

208. Which phenomenon is an interference pattern caused by the reflection of light between a spherical surface and an adjacent flat surface?
(a) Newton's rings
(b) Diamond rings
(c) Engagement rings
(d) Uranus rings

209. Which of the following is a technique for recording and reproducing an image of an object through the use of interference effects?
(a) photography
(b) tomography
(c) holography
(d) cartography

210. How many colours comprise white light?
(a) infinite (b) one
(c) three (d) seven

211. For which colour is the fringe width minimum?
(a) red (b) green
(c) violet (d) yellow

212. On which parameter intensity of light depends on?
(a) frequency
(b) wavelength
(c) amplitude
(d) velocity

213. In Fresenel's biprism experiment the coherent sources are obtained by
(a) reflection
(b) interference
(c) refraction
(d) total internal reflection

214. What is the path difference between the waves reaching the central fringe and bright fringe in Young's double slit experiment?
(a) zero (b) 3π
(c) 2π (d) 4π

215. Which are different types of emission spectrum?
(a) continuous spectrum
(b) line spectrum
(c) band spectrum
(d) all of the above

216. Which parameter determines the brightness of a light source sensed by an eye?
(a) light energy entering the eye
(b) wavelength of light
(c) total radiant flux entering the eye
(d) total luminous flux entering the eye

217. A photographic plate records sufficiently intense image when exposed with a 10 W source for 12 sec. How much time will be required with 12 W source?
(a) 8 sec (b) 9 sec
(c) 10 sec (d) 11 sec

218. Inverse square law for illuminance is valid for
(a) isotropic point source
(b) cylindrical source
(c) search light
(d) all types of sources

219. If the distance between a point source of light and a screen is doubled. The intensity will be
(a) four times the original value
(b) two times the original value
(c) half the original value
(d) one quarter of the original value

220. What is the unit of luminous efficiency of electric bulb?
(a) watt
(b) lux
(c) lumen
(d) lumen/watt

221. Candela is a unit of
(a) acoustic intensity
(b) electric intensity
(c) luminous intensity
(d) magnetic intensity

222. If the distance of a surface from light source is doubled, then the illuminance will become?
(a) 1/2 times (b) 2 times
(c) $\frac{1}{4}$ times (d) 4 times

223. As the wavelength is increased from violet to red, the luminosity
(a) increases continuously
(b) decreases continuously
(c) first increases then decreases
(d) first decreases then increases

224. Which of the following is the method used to measure the light speed in laboratory?
(a) Fizeau method
(b) Roemer Method
(c) Michelson method
(d) Foucault's method

225. A battery operated torch is adjusted to give parallel light beam. It produces illuminance of 60 lux on a wall 2 m away. The illuminance produced 3 m away is
(a) 20 lux (b) 40 lux
(c) 60 lux (d) 80 lux

226. Which instrument measures properties of light over a specific portion of the electromagnetic spectrum?
(a) Photometer
(b) Spectrometer
(c) Ammeter
(d) Lactometer

227. Persistence of vision is the phenomenon of the eye by which an afterimage is thought to persist for approximately one twenty-fifth of a second on the
(a) retina (b) heart
(c) mind (d) liver

228. Modern theoretical film runs at how many frames per second?
(a) 8 (b) 16
(c) 24 (d) 32

229. What is an elementary particle, the basic unit of light and all other forms of electromagnetic radiation?
(a) Phonon (b) Photon
(c) Neutron (d) Proton

230. Which portion of light has a wavelength in a range from 400 to 780 nm, with a frequency range of 405 to 790 Hz?
(a) infrared light
(b) visible light
(c) ultra-violet light
(d) none of the above

ANSWERS

1. a	2. b	3. b	4. b
5. b	6. b	7. c	8. a
9. b	10. a	11. a	12. a
13. c	14. c	15. b	16. b
17. a	18. b	19. c	20. a
21. d	22. a	23. d	24. a
25. a	26. b	27. a	28. a
29. d	30. d	31. a	32. c
33. b	34. c	35. d	36. a
37. c	38. b	39. a	40. b
41. a	42. b	43. a	44. b
45. b	46. d	47. a	48. a
49. d	50. c	51. a	52. a
53. b	54. b	55. d	56. c
57. b	58. d	59. d	60. b
61. b	62. b	63. d	64. b
65. d	66. d	67. b	68. d
69. c	70. c	71. c	72. d
73. d	74. c	75. b	76. a
77. a	78. c	79. b	80. a
81. d	82. a	83. b	84. a
85. c	86. b	87. c	88. a
89. d	90. b	91. d	92. d
93. b	94. a	95. c	96. c
97. c	98. d	99. d	100. a
101. b	102. c	103. d	104. a
105. a	106. c	107. c	108. c
109. a	110. d	111. c	112. d
113. b	114. d	115. c	116. c
117. d	118. a	119. d	120. c
121. b	122. c	123. c	124. b
125. c	126. a	127. c	128. c
129. c	130. c	131. b	132. d
133. a	134. d	135. a	136. c
137. b	138. c	139. a	140. c
141. a	142. b	143. a	144. d
145. a	146. a	147. c	148. c
149. b	150. d	151. c	152. b
153. d	154. c	155. c	156. b
157. b	158. b	159. a	160. b
161. d	162. b	163. d	164. c
165. c	166. c	167. b	168. a
169. b	170. b	171. c	172. c
173. b	174. c	175. a	176. a
177. a	178. a	179. b	180. c
181. b	182. d	183. c	184. a
185. a	186. b	187. b	188. d
189. b	190. a	191. c	192. c
193. b	194. b	195. b	196. a
197. a	198. a	199. a	200. c
201. d	202. b	203. b	204. a
205. b	206. c	207. b	208. a
209. c	210. d	211. c	212. c
213. c	214. d	215. d	216. d
217. c	218. d	219. d	220. d
221. c	222. c	223. c	224. d
225. c	226. b	227. a	228. c
229. b	230. b		